The Influence of Aspect Ratio Distributions on the Thermoelastic Properties of Short-Fiber Reinforced Composite Materials

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ABSTRACT

Short-fiber composites usually exhibit a distribution of fiber aspect ratios. Micromechanics models used for predicting the thermoelastic properties of short-fiber composite systems require a characteristic fiber aspect ratio value as an input parameter, usually taken to be the arithmetic average, \( \bar{a} \), of the fiber aspect ratio \( a \). Instead, this paper suggests that a more appropriate parameter is the square-root of the arithmetic average of the reciprocal of the square of the aspect ratio, \( \sqrt{\frac{1}{\bar{a}^2}} \). Models are presented which relate this measure of the aspect ratio distribution to the thermoelastic behavior.

INTRODUCTION

The central problem of a micromechanical analysis is the specification of an internal stress (or strain) field consistent with the external field imposed on the macroscopic body. The load-deformation field acting on a homogeneous macroscopic body can be obtained from the specification of the load-deformation pattern imposed on the boundaries of the body. The resulting internal surface tractions and deformations acting on the internal boundaries of the microscopic phase regions are not so readily obtained. The internal stress-strain fields of a heterogeneous system are locally influenced by (i) the relative magnitudes of the constituent properties of the components, (ii) the relative shape and orientation of the phase regions, and (iii) the packing geometry of the phase regions.

An important problem arises in specifying the appropriate features of the microstructure to serve as the dominant structural descriptors. A focus on the average response characteristics of a heterogeneous material will submerge localized variations in the microstructure and associated internal fields so that any distribution of these quantities is replaced by statistically equivalent averages and/or higher moments of the structural distributions. Although of lesser importance for continuous fiber reinforced composites, the identification of proper statistical descriptors which are accessible to experimental evaluation becomes a critical issue for short-fiber reinforced composites.
Wu and McCullough [1] developed a general bounding theory for the elastic behavior of heterogeneous materials based on the variational methods developed by Hashin and Shtrikman [2-4], and by Walpole [5,6]. This theory formulated the variational problem in terms of deviatoric quantities with respect to an arbitrary reference system. Through appropriate choices of the reference system, the general bounding formulation was shown to reduce to several different bounding formulations.

Eduljee [7] extended the Wu and McCullough formulation for dispersed short fiber systems. He derived the lower bound (resin as the reference phase) on the effective elastic properties, \([C_{ib}]^\ast\), of a dispersed short fiber composite as:

\[
[C_{ib}]^\ast = [C_m] + \{v_f \langle [M_f] \rangle [H_f] + v_m \}^{1/2} \{v_f \langle [M_f] \rangle \}
\]  

where the subscripts \(m\) and \(f\) denote the matrix and fiber phases, respectively, \(v_m\) and \(v_f\) are the volume fractions of the matrix and fiber phases, respectively, \([C_m]\) is the stiffness of the matrix phase and \([E^o]\) is a modified Eshelby's tensor [1], and is a function of the fiber aspect ratio \(a\) and the reference phase. For the lower bound case, \([E^o]\) is evaluated with the matrix as the reference phase. The quantity \([M_f]\) is defined in terms of deviatoric quantities as:

\[
[M_f] = ([H_f] - [E^o])^{-1}
\]  

\[
[H_f] = ([C_f] - [C_m])^{-1}
\]

The angle brackets, \(\langle q \rangle\) signify the configurational average of \(q\) (a second rank tensor, or a vector). Configurational averaging includes both orientation averaging as defined in [8], as well as averaging over the fiber aspect ratio distributions.

The corresponding quantities for the upper bound (fiber as the reference phase) on the elastic properties, \([C_{ub}]^\ast\), are given by:

\[
[C_{ub}]^\ast = [C_f] + \{v_m \langle [M_m] \rangle [H_m] + v_f \}^{1/2} \{v_m \langle [M_m] \rangle \}
\]  

where

\[
\langle [M_m] \rangle = ([H_m] - \langle [E^o] \rangle)^{-1}
\]

For the upper bound case, \([E^o]\) is evaluated with the fiber as the reference phase.

The focus of the current treatment is on the identification of the appropriate statistical descriptor characterizing the aspect ratio distribution in short fiber composites. Accordingly, attention will be restricted to aligned inclusions which can be subsequently subjected to orientation averaging. The following arguments will show that for aspect ratios \(a > 20\), the averages over an aspect ratio distribution can be reasonably approximated by replacing the