A 3D IEM for Compressible Wing Flows With and Without Shocks
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ABSTRACT

An integral equation (or called field-panel, field-boundary element) scheme for solving the full-potential equation for incompressible and compressible flows with and without shocks has been developed. The full-potential equation has been written in the form of the Poisson's equation. Compressibility has been treated as non-homogeneity. The integral equation solution in terms of velocity field is obtained by the Green's theorem. The solution consists of wing (or a general body) surface (boundary elements) integral term(s) of vorticity/source distribution(s), wake surface (boundary elements) integral term(s) of free-vortex sheet(s), a volume (field-elements) integral term of compressibility over a small limited domain around the source of disturbance, and a shock surface (boundary elements) integral term of source distributions. Solution is obtained through an iterative procedure for non-linear compressible flows. To be consistent with the mixed-nature of transonic flows, the Murman-Cole type-difference scheme is used to compute the derivatives of the density. The present scheme is applied to flows around a rectangular wing with circular-arc section at incompressible, high-subsonic and transonic flow conditions.

Key Words: integral equation method, full-potential equation, subsonic and transonic wing flows.

INTRODUCTION

The finite-difference method (FDM) and finite-volume method (FVM) for solving transonic flows have been well developed during the past twenty years. Although the Navier-Stokes equation formulation for the transonic flow computations has been understood as the best model and the FDM and FVM are successful in dealing with transonic flows, the computation of the unsteady Navier-Stokes equations over complex three-dimensional configurations is very expensive, particularly for time-accurated unsteady flow computations. There are also major technical difficulties in FDM and FVM for generating suitable grids for complex three-dimensional aerodynamic configurations.
The experience has shown that rather accurate solutions can be obtained for many transonic flows using the inviscid modeling of the full-potential equation. For transonic flows without strong shocks and massive separations, the full-potential equation is an adequate approximation to the Navier-Stokes equations. The integral equation method (IEM) for the potential equation is an alternative to the FDM and FVM. Moreover, the IEM has several advantages over the FDM and FVM. The IEM involves evaluation of integrals, which is more accurate and simpler than the FDM and FVM, and hence a coarse grid (field-elements) can be used in IEM. The IEM automatically satisfies the far-field boundary conditions and therefore only a small limited computational domain is needed. The IEM does not suffer from the artificial viscosity effects as compared to FDM and FVM for shock capturing in transonic flow computations. The generation of the three-dimensional grid for complex configuration is not difficult in the IEM, since the mapping from physical plane to computational plane is not required.

Integral equation methods for transonic flows have been developed by several investigators\textsuperscript{1-18} during the past few years for steady airfoil, wing and aircraft configurations and unsteady airfoils and wings. In the present paper a method for computing general steady 3-D flows, which is an extension of 2D method of Ref.10, is presented along with simple numerical examples to demonstrate the capability and the potential of the present IE scheme for incompressible, subsonic and transonic flow computations. The shock-fitting technique is applied to the present transonic flow calculations, so that a coarse grid can be used.

**FORMULATION**

**Governing Equations**
The non-dimensional steady full-potential equation is given by:

\[ \nabla^2 \Phi = G \]  
\[ G = -\frac{\nabla \rho}{\rho} \cdot \vec{V} \]  
\[ \rho = [1 + \frac{\kappa - 1}{2} (1 - |\vec{V}|^2)]^{\frac{1}{\kappa - 1}} \]

where the characteristic parameters, \( \rho_\infty \), \( a_\infty \) and \( l \) have been used; \( a \) is the speed of the sound, \( \rho \) the density, and \( l \) the wing surface panel length; and \( \Phi \) is the velocity potential (\( \vec{V} = \nabla \Phi \)), \( G \) the compressibility, and \( \kappa \) the gas specific heat ratio.

**Boundary Conditions**
The boundary conditions are surface no-penetration condition, Kutta condition, infinity condition, and wake kinematic and dynamic conditions. They are described as follows:

\[ \vec{V} \cdot \vec{n}_g = 0 \quad \text{on} \quad g(\vec{r}) = 0 \]