PARAMETRIC IDENTIFICATION OF MECHANICAL STRUCTURES:
GENERAL ASPECTS OF THE METHODS USED AT THE LMA

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ABSTRACT

The accent is placed on the localization of the dominant modelling errors by a sub-space method. Other points also discussed concern the preparation of tests for a parametric identification: optimal selection of the pick-up positions with respect to: the reconstitution of the unobserved degrees of freedom; the orthogonality and the numerical conditioning of the Jacobian matrix.

OBJECTIVES

This paper investigates the problem of the improvement in the quality of predictive models used in calculating the dynamic behavior of mechanical structures. The following assumptions are made: the structures are linear elastodynamic; they lead to mathematical models which are "simple" in the sense that they are governed by state matrices which are strictly diagonalizable (no Jordan blocks; no principal vectors). The case of non self-adjoint and "non simple" structures can be treated with analogous although more complicated methods. The proposed methods exploit data which are stationary with respect to time: eigensolutions and frequency responses.

It is sought to correct the state matrices, $M^a$, $B^a$, $K^a \in \mathbb{R}^{n \times n}$, symmetric positive definite, representing the initial estimation (Finite Elements) using the measured dynamic behavior of the real physical structure. This parametric identification problem is reduced to a linear or non-linear parametric optimization problem with constraints.

The technical applications envisaged are:
* The improvement of the quality of predictive models for the global (displacement field) or local (stress field) behavior in the presence of exterior excitations or new structural configurations (Ex: dynamic sub-structuring);
* Improvement of the data bases in the case of sub-structures or of boundary conditions which are difficult to modelize (Ex: ground-structure interaction);
* Detection of the modelization errors and aid in a partial analytical remodelization;
* Surveillance and localization of structural failures (Civil engineering examples: Dams, Cooling towers).

The aspect studied here is: The parametric correction of the initial estimation. In the case where this leads to incoherent results, a complete reformulation of the initial estimation must be performed. This aspect is not studied here.
The "localization of dominant errors" is to be understood in the following sense. The reduction of the "distances" between the predicted behavior of the model and the partially observed behavior of the structure is often reduced to a problem of minimizing a cost function constituted of quadratic forms. The necessary condition for the minimization of this function with or without local linearization is thus reduced to the resolution of a local problem which is linear and non homogeneous, either under or over determined and having real matrices of the form: \[ Ax = b \] (1)

where : \( b \) is the representation of the distances, 
\( x \) the local parametric correction vector (in the simplest case).

The resolution of (1) by pseudo-inverse generally does not lead to a physically satisfying solution. Schematically :
- if (1) is under-determined, a solution \( \hat{x} \) of minimal norm privileges the columns of \( A \) having dominant norms. This solution has no physical significance ;
- if (1) is over-determined, a solution \( \hat{x} \) of minimal error norm leads to dominant components \( \hat{x}_i \) of \( \hat{x} \) which are associated with columns of \( A \) having small norms.
- a solution \( \hat{\hat{x}} \) obtained form a regularization (for example of the type Tikhonov [1]) leads to a intermediate solution between the 2 preceding cases.

In all these cases, the solution \( \hat{x} \) does not allow the poorly modeled columns of \( A \) (and ultimately the sub-domains of the structure) to be detected (except if \( A \) is an orthogonal matrix). In terms of the concept of attempting to localize the physical modelization errors it is proposed to solve (1) in a qualitative manner with the objective of detecting the smallest number of parameters contributing in a dominant way to the distance "\( b \)".

**LOCALIZATION BY THE SUB-SPACE METHOD [2;3]**

The localization of the dominant modelling errors in the estimation \( M^a \); \( K^a \) (and eventually \( B^a \)) is reduced to a selecting among the columns of \( A \) those which are the closest to the direction defined by \( b \). The global quality of this localization depends on the projection of the vector \( b \) in the sub-space whose basis vectors are the columns of \( A \).

The first useful operation consists thus in evaluating the rank of the matrix \( A \) and that of the augmented matrix \( \tilde{A} = [A; b] \). If the ranks of these 2 matrices are the same, then the necessary condition for the localization is satisfied. If \( R(\tilde{A}) > R(A) \), a perfect localization will be impossible.

From a numerical point of view, the following procedure is followed. Let \( \sigma_i \) be the \( i^{th} \) singular value of \( A \) and : \( r_i = \sigma_i / \sigma_1 \). \( R(A) \) is defined to be equal to the number of quantities \( r_i \) superior to a given tolerance \( T \). After determination of the singular values of \( \tilde{A} \) and \( A \), the evolution of \( r_i \) as a function of \( i \) can be plotted for \( \tilde{A} \) and \( A \) (Fig. 1).

![Figure 1a](image1a.png) ![Figure 1b](image1b.png)