MEASUREMENT OF COMPLEX MODULI OF COMPOSITE MATERIALS
AND DISCUSSION OF SOME RESULTS

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ABSTRACT
This paper describes a new method for the characterization of the engineering constants of linear visco-elastic orthotropic materials. The method is an application of the principle of mixed numerical/experimental techniques: measured responses are compared to responses calculated from a numerical model and the parameters of the numerical model are then updated until the two types of responses match. For the technique discussed in this paper, the parameters of the numerical model are the complex engineering constants and the measured and calculated responses are vibrational data. Emphasis is put on the characterization of the damping behaviour, the discussion of all possible error sources and some interesting results.

INTRODUCTION
The method discussed in this paper was especially developed to meet the requirements of engineers working and designing with composite materials. It is a very fast and non-destructive technique that combines the characterization of both the elastic behaviour and the damping behaviour of the material. It provides global and representative results since it is based on the measurement of global quantities on relatively large test specimen.

The reason why the technique should also be able to characterise material damping is that, in composite structures, very little energy is dissipated within the mechanical joints, as for example in metal structures. That is because the number of joints in composite structures is smaller and because they are generally of the adhesive type. So, the inherent damping of the material itself plays a very important role and a simple and fast method to determine this material damping in an experimental way is needed.
The method can be applied to all materials that satisfy the following three conditions:

- linear visco-elastic material behaviour;
- orthotropic material properties;
- available under the form of thin plates (Love-Kirchhoff hypothesis).

When these three conditions are satisfied, the mechanical behaviour of the material can be characterized by four complex engineering constants:

\[
\begin{align*}
E_1^* &= E_1(1 + i \cdot \tan(\delta_{E_1})) : \text{Complex Young's modulus (1-direction)} \\
E_2^* &= E_2(1 + i \cdot \tan(\delta_{E_2})) : \text{Complex Young's modulus (2-direction)} \\
\nu_{12}^* &= \nu_{12}(1 + i \cdot \tan(\delta_{\nu_{12}})) : \text{Complex Poisson's ratio} \\
G_{12}^* &= G_{12}(1 + i \cdot \tan(\delta_{G_{12}})) : \text{Complex inplane shear modulus}
\end{align*}
\]

The real parts of these engineering constants are called the elastic moduli since they characterize the elastic behaviour, while the imaginary parts are called the loss moduli and they characterize the damping of the material.

The technique for the determination of the purely elastic material properties was developed first and is now a very reliable and widely used technique [1]. The technique for the determination of the visco-elastic material properties can be seen as an expansion, since it makes use of the same numerical model and the same experimental set-up.

**THEORETICAL BACKGROUND**

The equilibrium equation of a single degree of freedom system with viscous damping (see figure 1):

\[
m \ddot{x} + c \dot{x} + kx = 0
\]

yields the complex stiffness and the loss tangent:

\[
k^* = k + i \cdot \omega \delta = k(1 + i \cdot \tan(k))
\]

where: \( \tan(k) = \omega \delta / k \)

![Figure 1. Single degree of freedom system with viscous damping](image-url)