1. Introduction

The knowledge of artificial composite materials (French acronym - *Science de Matériaux ARTificiels*) reaches a prehistory. Architecture, textile craftsmanship, jewellery have used such materials extensively. For example, in mid-way between Pultusk and Warsaw, in Wieliszew in 1992, in an archaeological excavation, a V-IV century dish made of clay ceramics with 3 brass circles inserted was found.

*Intelligent* (English "smart") materials denote objects which are manufactured in such a way that they perform certain planned actions. A number of smart materials are related to discovery of a piezoelectric phenomenon by the brothers Jacques and Pierre Curie in 1880, [1]. A composite known as a "Curie strip (bilame de quartz)" invented in 1889 and consisting of two X-cut quartz plates cemented together with polarities opposed, and vibrating in a flexural mode, is one of such smart materials, [2, 4, 5]. The other composite material is known as Langevin’s quartz-steel "sandwich", [3, 5]. It has the form of a few millimeters thick quartz plate cemented between two massive slabs of steel that serve as electrodes in a transducer.

The aim of this contribution is to analyze the formulae for macroscopic moduli of layered piezoelectric composites in terms of anisotropy of the layers. We limit ourselves to the study of a composite made of the periodically arranged quartz layers with different directions of crystal axes in each layer.

First, by using the general homogenization formulae derived in [7], cf. also [8-11], the effective elastic, piezoelectric and dielectric moduli for a microscopic layered composite are obtained. These formulae are derived in a particular reference frame. Thus a natural question arises: how could we obtain the effective moduli in arbitrary direction of lamination. Moreover, a macroscopic behaviour of layered piezoelectric composites, in which
anisotropy axis of each layer constituting the basic cell do not coincide, is
also of some interest. By choosing particular piezoelectric materials it is
shown that depending on the anisotropy axis of layers, the effective moduli
differ from one composite to another, though the layers are made of the
same materials.

2. Basic relations

Let \( \Omega \subset \mathbb{R}^3 \) be a bounded, sufficiently regular domain and \((0, \tau)(\tau > 0)\)
– a time interval. The elastic, piezoelectric and dielectric moduli are de-
noted by \( c_{ijkl}, g_{ijkl} \) and \( \epsilon_{ij} \), respectively; \( \rho \) is the density. We identify \( \Omega \)
with the underformed state of the piezoelectric composite with a micropе-
periodic structure. Thus for \( \varepsilon > 0 \) the material functions just introduced are
\( \varepsilon Y \)-periodic, where \( Y = (0, Y_1) \times (0, Y_2) \times (0, Y_3) \) is the so-called basic cell. We write
\[
c_{ijkl}(x) = c_{ijkl}(\frac{x}{\varepsilon}), \quad g_{ijkl}(x) = g_{ijkl}(\frac{x}{\varepsilon}), \quad \epsilon_{ij}(x) = \epsilon_{ij}(\frac{x}{\varepsilon}), \quad \rho(x) = \rho(\frac{x}{\varepsilon}),
\]
where \( x \in \Omega \) and the functions \( c_{ijkl}^{\varepsilon}, g_{ijkl}^{\varepsilon}, \) etc are \( \varepsilon Y \)-periodic, where \( \varepsilon > 0 \)
is a small parameter.

For a fixed \( \varepsilon > 0 \) the basic relations describing a linear piezoelectric
solid with the microperiodic structure are:

(i) **Field equations**

\[
\sigma_{ij,j} + b_i^\varepsilon = \rho \ddot{u}_i^\varepsilon, \quad D_i^\varepsilon = 0 \quad \text{in} \quad \Omega \times (0, \tau).
\]

(ii) **Constitutive equations**

\[
\sigma_{ij}^{\varepsilon} = c_{ijkl}^{\varepsilon} \varepsilon_{kl}(u^\varepsilon) - g_{ijkl}^{\varepsilon} E_k(\varphi^\varepsilon), \quad D_i^\varepsilon = g_{ikl}^{\varepsilon} \varepsilon_{kl}(u^\varepsilon) + \epsilon_{ik}^{\varepsilon} E_k(\varphi^\varepsilon).
\]

(iii) **Geometrical relations**

\[
e_{kl}(u^\varepsilon) = u_{(k,l)}^\varepsilon = \frac{1}{2}(u_{k,l}^{\varepsilon} + u_{l,k}^{\varepsilon}), \quad E_k(\varphi^\varepsilon) = -\varphi_k^{\varepsilon}.
\]

Here \((\sigma_{ij}^{\varepsilon}), (u_i^\varepsilon), (b_i^\varepsilon), (\rho^\varepsilon), (E_i^{\varepsilon}), \varphi \) and \((D_i^{\varepsilon})\) are the stress tensor, the
displacement vector, the body force vector, the mass density, the electric
field vector, the electric potential and the electric displacement vector, re-
spectively. Moreover \( b^\varepsilon \) is \( \varepsilon - Y \) periodic. The tensors of material functions
satisfy the usual symmetry conditions
\[
c_{ijkl}^{\varepsilon} = c_{mlij}^{\varepsilon} = c_{ijlm}^{\varepsilon} = c_{jimn}^{\varepsilon}, \quad g_{ijkl}^{\varepsilon} = g_{klij}^{\varepsilon}, \quad \epsilon_{ij}^{\varepsilon} = \epsilon_{ji}^{\varepsilon}.
\]

We make the usual assumption: there exists a constant \( \alpha > 0 \) such that for
almost every \( x \in \Omega \), the following conditions are satisfied:
\[
c_{ijmn}^{\varepsilon}(x)e_{ij}e_{mn} \geq \alpha \left| e \right|^2, \quad \epsilon_{ij}^{\varepsilon}(x)a_ia_j \geq \alpha \left| a \right|^2,
\]