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INPUT PRICE, QUANTITY, AND PRODUCTIVITY INDEXES FOR A REVENUE-CONSTRAINED FIRM

by
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2.1 INTRODUCTION

This paper develops the micro-economic theory of input price and quantity indexes and input-based productivity indices for a revenue-constrained firm. Generally speaking, a firm transforms inputs into outputs. When priced, outputs generate revenue and inputs incur cost. It is customary to (partially) model the economic behavior of a firm as cost minimization. Then the input price index is calculated as the ratio of minimum costs under two different price regimes. However, in doing so, one has to condition on certain output variables. One route is to take the output quantities as the conditioning variables. The firm's objective is then conceived as the production of a vector of output quantities with minimal cost. This leads to a theory of input price indexes which is, except for the dimension of the vector of conditioning variables, isomorphic to the theory of the cost-of-living index for consumers.

*The author thanks Robert G. Chambers and R. Robert Russell for their comments on a previous version. The views expressed in this paper are those of the author and do not necessarily reflect the policies of Statistics Netherlands.

R. Färe et al. (eds.), Index Numbers: Essays In Honour of Sten Malmquist
Fisher (1995) considered this approach, except for the case of a single-output firm, as being too narrow. According to his view "the demand conditions that a competitive firm takes as given are not represented by a vector of fixed output quantities but by a vector of fixed output prices at which the firm can sell." Thus a more realistic route is to condition on output prices, and to take the firm's objective as the attainment of a target revenue with minimal input cost. This leads to the theory of the revenue-constrained firm, as developed by Färe and Grosskopf (1994). The applicability of this theory is, however, not restricted to single firms. Sidestepping the aggregation issue, Fisher (1995) uses the revenue-constrained firm as a model for a small, fully open economy which trades outputs on world markets at fixed prices.

Having pointed out the significance of the present topic, we proceed to describe the plan of this paper. Section 2 briefly reviews a number of concepts from duality theory. We limit ourselves to those concepts that play a role in the ensuing sections. Section 3 is devoted to the indirect distance function and the indirect cost function. Both sections build heavily on Färe and Primont (1995). Using the concepts introduced in Section 3, Section 4 formally defines the indirect input price and quantity indexes. Their properties will be shown, and we discuss under which assumptions the theoretical indexes can be approximated by or calculated as Fisher or Törnqvist index numbers. Section 5 turns to the definition of productivity indexes, and shows under which assumptions the theoretical productivity indexes transform into compositions of statistical index numbers. By way of conclusion, Section 6 summarizes the main results in non-technical language.