

## MULTI-SCALE ASPECTS IN LINEAR AND NONLINEAR ESTIMATION AND CONTROL

G. STEPHANOPOULOS, O. KARSLIGIL AND M.S. DYER

*Department of Chemical Engineering  
Massachusetts Institute of Technology  
Cambridge, MA 02139*

Multi-scale models of processing systems offer an attractive alternative to models defined in the time- or frequency-domain. They are the outgrowth of a series of developments which came about with the advent of the wavelet decomposition for the analysis of discrete signals. Multi-scale models are defined on dyadic or higher-order trees. The nodes of such trees are used to index the values states, inputs and outputs, modeling errors, and measurement errors. These values are localized in both time and scale (range of frequencies), and thus they offer a hybrid domain that is particularly conducive for estimation and control problems. In this paper we introduce a formal framework for the formulation of multi-scale models on trees, which are consistent with their time-domain counterparts. Such models lead to a multi-scale control theory and the definition of the corresponding transfer functions, stability, controllability, and observability concepts for systems on trees. Fusion of control actions and measurements at different rates as well as their implications on the controllability and observability of dynamic systems are also examined. Of particular significance is the issue of closure between the models in the time- and the time-scale domains, which constrains the type of physico-chemical processes that can be modeled on a tree.

Based on these developments, we then proceed to address the solution of certain basic tasks in systems engineering, such as; simulation of linear systems, optimal control, and state estimation with optimal fusion of measurements. It is shown that the multi-scale models lend themselves nicely to parallelizable computations, which can produce algorithms of substantially lower computational load. Multi-scale Model Predictive Control is subsequently formulated and the ensuing estimation and optimal control sub-problems on trees are examined. The time-scale localization of the states, inputs, outputs and errors, in a multi-scale MPC allows a more explicit selection of the design specifications for the MPC formulation. Furthermore, it is shown that the reduced computational load offered by the parallelizability of many computational tasks, leads to a re-examination of the ways that classical MPC formulations are addressed in the time-domain, and thus a re-examination of how closed-loop stability, constraint satisfaction, horizon determination, and others, can be resolved. Finally, the paper discusses several tentative ideas and suggestions on how multi-scale aspects can be extended to handle nonlinear systems.

## 1. INTRODUCTION

Control of nonlinear processes through Model-Predictive Control (MPC) strategies, raises a series of formidable problems, the source for most of which can be traced back to the following two distinctive features [37].

- (a) The open-loop optimal control policy, computed at each step, must be globally optimum if it is to be admissible.
- (b) The closed-loop stability requirement imposes a “stability constraint”, i.e. an equality constraint,  $x(t=T) = 0$ , on the terminal state, which can be achieved only asymptotically in time.

In addition, inequality constraints on inputs impose constraints on what states can be reached by the allowed control actions, while inequality constraints on outputs have an impact on the closed-loop stabilization of a process. Progress has been slow and only in fairly specialized classes of models (not Physico-chemical systems), such as the “finite response” systems, feasibility of control policies and terminal conditions for closed-loop stability can be readily established.

However, the *raison-d'être* for nonlinear process control research and technology must be founded on the answer of the following simple question: “What is to be gained by nonlinear control over what is accomplished by linear control”? The answers have not been very convincing. Nevertheless, substantive progress in nonlinear process control and development of credible technology may change the “cultural” inhibitions of process designers and allow the deployment of processes, which “must” possess nonlinear control systems.

Given the formidable difficulties, offered by the general nonlinear control problem, and its “cultural” affinity with the linear control tradition (e.g. MPC was developed as a control strategy for linear constrained systems, why should it be the correct paradigm to tackle nonlinear control systems?), in this paper we will attempt to explore an alternative framework to the classical linear systems theory, in an attempt to establish a formalism that can solve a wide class of control problems. Thus, instead of tackling nonlinear control problems directly, we will examine whether the multi-scale systems theory, the foundation of the methods in this paper, can solve a broader class of problems through linear methods, reducing in the process the need for nonlinear control.

### *The Need for Scale and Time Localization of Processes and Measurements*

It is broadly accepted that physical phenomena occur at different time scales. However, it is not clear how to systematically incorporate this knowledge in the generation of adequate process models, or how to use it for the solution of some basic process engineering problems, e.g. control, estimation, diagnosis. The conventional models, causal and explicit, provide a convoluted representation of physics at various scales and hamper engineering analysis and interpretation. Furthermore, process models used in