SHEAR BAND LOCALIZATION IN FLUID-SATURATED GRANULAR ELASTO-PLASTIC POROUS MEDIA

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1. Introduction

Shear band localization phenomena as appearing, e.g., in the classical base failure problem of geotechnical engineering, occur in non-saturated and saturated soils as a result of local concentrations of plastic solid strains (de Borst, 1991 a; Schrefler et al., 1995, 1996). The present contribution concentrates on the saturated case, where the elasto-plastic deformations of the granular soil can be described by use of a macroscopic continuum mechanical approach within the well-founded framework of the Theory of Porous Media (TPM), compare, e.g., the work by Bowen (1982), de Boer and Ehlers (1986), Ehlers (1989, 1993) and de Boer (1996).

In the present article, the TPM formulation of the skeleton material is extended by micropolar degrees of freedom in the sense of the Cosserat brothers (de Borst, 1991 b; Steinmann, 1994). Proceeding from two basic assumptions, material incompressibility of both constituents (skeleton material and pore-fluid) and geometrically linear solid deformations, the non-symmetric effective skeleton stress and the couple stress tensor are determined by linear elasticity laws. In the framework of the ideal plasticity concept applied to cohesive-frictional soils, the plastic yield limit is governed by a smooth and closed single-surface yield function together with non-associated flow rules for both the plastic strain rate and the plastic rate of curvature tensor. Fluid viscosity is taken into account by the drag force.

The inclusion of micropolar degrees of freedom, in contrast to the usual continuum mechanical approach to the TPM, allows, on the one hand, for the determination of the local average grain rotations and, on the other hand, additionally yields a regularization effect on the solution of the strongly coupled system of governing equations when shear banding occurs. The numerical example exhibits the base failure problem induced by elasto-plastic consolidation. The computations are carried out by use of finite element discretization techniques.
2. Governing Equations

In the framework of the Theory of Porous Media, fluid-saturated porous solid skeleton materials are considered as a mixture of immiscible constituents \( \varphi^\alpha \) with particles \( X^\alpha (\alpha = S: \text{solid skeleton}; \alpha = F: \text{pore-fluid}) \), where at any time \( t \), each spatial point \( x \) of the current configuration is simultaneously occupied by material points \( X^\alpha \) of all constituents \( \varphi^\alpha \) (superimposed continua). These particles proceed from different reference positions \( X_\alpha \) at time \( t_0 \). Thus, each constituent is assigned an individual motion function

\[
x = x_\alpha (X_\alpha, t).
\]

The volume fractions

\[
n^\alpha = n^\alpha (x, t)
\]

are defined as the local ratios of the constituent volumes \( v^\alpha \) with respect to the bulk volume \( v \). Thus, the saturation condition yields

\[
n^S + n^F = 1.
\]

Associated with each constituent \( \varphi^\alpha \) is an effective (realistic or material) density \( \rho^{\alpha R} \) and a partial (global or bulk) density \( \rho^{\alpha} \). The effective density \( \rho^{\alpha R} \) is defined as the local mass of \( \varphi^\alpha \) per unit of \( v^\alpha \), whereas the bulk density \( \rho^{\alpha} \) exhibits the same mass per unit of \( v \). The density functions are related by

\[
\rho^\alpha = n^\alpha \rho^{\alpha R}.
\]

Proceeding from (4), it is obvious that the property of material incompressibility of any constituent \( \varphi^\alpha \) (defined by \( \rho^{\alpha R} = \text{const.} \)) is not equivalent to global incompressibility of this constituent, since the partial density functions can still change through changes in the volume fractions \( n^\alpha \).

It follows from (1) that each constituent is assigned its own velocity field. Thus, by use of either the \textit{Lagrangean} or the \textit{Eulerian} description,

\[
x'_\alpha = \frac{\partial x_\alpha (X_\alpha, t)}{\partial t}, \quad \dot{x}_\alpha = x_\alpha (x, t).
\]

Assume that \( \Gamma \) is any arbitrary, continuous and sufficiently often continuously differentiable function of \( (x, t) \). Then, the material time derivatives of \( \Gamma \) corresponding to the individual motion functions of \( \varphi^\alpha \) are given by

\[
(\Gamma)'_\alpha = \frac{\partial \Gamma}{\partial t} + \text{grad} \Gamma \cdot x'_\alpha.
\]

Therein, the operator \textquotedblleft grad \((\cdot)\)\textquotedblright\) characterizes the partial derivative of \((\cdot)\) with respect to the position vector \( x \) of the actual configuration.

Concerning the problem under study, it is convenient to consider the solid motion in the frame of the \textit{Lagrangean} description by the introduction of the displacement vector

\[
u_S = x - X_S,
\]