NONLINEAR ANALYSIS OF INTERFACIAL CRACKS

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Abstract

Asymptotic analysis is developed for the problem of a plane strain crack lying on the interface of an elastic-plastic (or elastic-creeping) material and a rigid substrate. The nonlinear plastic (creep) response is taken to follow a power-law hardening relation. In contrast to other recent analyses, we have found asymptotic solutions for a continuous variation of crack-tip mode-mix that agree well with full-field solutions. These crack-tip displacements and stresses are variable-separable in polar coordinates \( r \) and \( \theta \) and exhibit a singularity in stress of \( \sigma \propto r^{-1/(n+1)} \) as \( r \to 0 \), where \( n \) is the hardening exponent. The angular variations of these asymptotic fields have been calculated using a finite difference scheme. Unlike the full range of mixed-mode solutions that exist for a homogeneous crack in a power-law hardening material, there appears to be a narrow range around the pure tensile mode for which solutions do not exist. That latter range increases somewhat as the hardening exponent \( n \) increases.

1. Introduction

The performance of structural materials is often determined by the response of interfaces between structural components, such as cracking of grain boundaries in polycrystals and fiber/matrix interfaces in composite materials. This paper addresses specifically the mechanics aspect of interfacial fracture. Rice (1988) and Hutchinson and Suo (1992) have provided an extensive review of work dating back to the late 1950's on the subject of fracture along interfaces between dissimilar linearly elastic materials. The crack tip fields of a linear system are found to be oscillatory, except in general when both isotropic materials are incompressible with Poisson's ratio \( v = 1/2 \). In contrast, recent investigations have been inconclusive regarding the structure of the crack tip fields for stationary crack problems at nonlinear bimaterial interfaces.

In recent analyses of interfacial cracks lying between a power-law hardening material and a rigid substrate, Wang (1990), Champion and Atkinson (1991) and Sharma and Aravas (1993) attempted to find crack-tip solutions that are separable in polar coordinates \( (r, \theta) \) and exhibit the singularity \( \sigma \propto r^{-1/(n+1)} \) as \( r \to 0 \). This stress singularity in the leading order term of the asymptotic expansion can be deduced from \( J \)-integral analysis and, consequently, is the same as the Hutchinson-Rice-Rosengren
solutions for cracks in homogeneous materials (Hutchinson, 1968; Rice and Rosengren, 1968). However, as opposed to the stationary homogeneous crack problem which yields mixed-mode solutions varying continuously between pure mode 1 and 2 (Shih, 1974), only a single mode-mix was obtained for each hardening exponent \( n \), in spite of the different numerical schemes used in Wang (1990), Champion and Atkinson (1991) and Sharma and Aravas (1993). The interfacial crack solutions presented in those papers are near mode 1, i.e., tensile, with some (positive) shear.

Several full-field analyses also exist in the literature for elastic/plastic interface cracks under small-scale yielding conditions. Shih and Asaro (1988) and Sharma and Aravas (1993) considered an interfacial crack between a power-law material with \( \nu = 0.3 \) and a rigid substrate under applied mode I and mixed-mode loading conditions. In a separate study, Shih and Asaro (1989) investigated an interfacial crack between a power-law material with \( \nu = 0.3 \) and an elastic substrate under mixed-mode loading. Generally, the crack-tip fields from these finite element analyses do not agree with the solutions obtained from the asymptotic analysis discussed previously. However, some components of the near-tip stresses, particularly the shear stresses, appear to attain the singularity \( \sigma \propto r^{-1/(n+1)} \).

In this paper, we analyze the interfacial crack problem, with the emphasis on the spectrum of separable mixed-mode asymptotic solutions. The stationary interfacial crack configuration considered, as depicted in Fig. 1, is a semi-infinite crack lying between a power-law hardening material and a rigid substrate and loaded under plane strain conditions. The deformable medium occupies the region \( 0 \leq \theta \leq \pi \), and the crack face is taken to be open and, therefore, traction free. Consequently, the boundary conditions satisfied by the asymptotic fields are:

\[
\begin{align*}
    u_r(r,0) &= u_\theta(r,0) = 0 \\
    \sigma_{r\theta}(r,\pi) &= \sigma_{\theta\theta}(r,\pi) = 0
\end{align*}
\]

We will make use of the mode-mix parameter, \( M \), defined as (Shih, 1974)

\[
M = \frac{2}{\pi} \tan^{-1} \left( \lim_{r \to 0} \frac{\sigma_{\theta\theta}(r,0)}{\sigma_{r\theta}(r,0)} \right)
\]