In the current discussion on philosophy of mathematics some do as if systematic foundational work supported an exclusive alternative between Platonism and Constructivism; others do as if such mathematical and logical research were deeply misguided and had no bearing on our understanding of mathematics. Both attitudes prevent us from grasping insights that underlie such work and from appreciating significant results that have been obtained. In consequence, they keep us from turning attention to the task of understanding the role of accessible domains for foundational theories and that of abstract structures for mathematical practice.

This twofold task derives from a probing perspective that takes seriously traditional epistemological concerns, but that does not respect time-honored boundaries drawn for philosophical convenience. It will be approached mainly through work that has been done during the last seventy years on versions of Hilbert's Program. Such an avenue may be surprising, because the stand that was taken in the foundational discussion by Hilbert, Bernays, and their collaborators is widely perceived as extremely narrow and technical. So I will give in section 2 a revisionary description of Hilbert's Program and sketch in section 3 some results that have been obtained within a general reductive program.

A prerequisite for Hilbert's Program is the effective or formal presentation of mathematical thought. Gödel took his incompleteness theorems as refuting any form of "pure formalism", in particular the variety (he thought to be) underlying Hilbert's Program. The discussion of Gödel's reflections on this issue, in section 4, will lead me to focus on two aspects of mathematical experience. The first is the quasi-constructive aspect, and it has to do with accessible domains; the second is the conceptional aspect, and it deals with axiomatically characterized abstract struc-
tures. These two aspects are discussed in sections 5 and 6. In the seventh and final section I come back to the question of "mechanizing" mathematical thought and contrast Turing's views with Gödel's.

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Are the results of contemporary proof theory significant for the foundational concerns that motivated Hilbert's Program, and are those concerns connected to insightful reflections on the nature of mathematics? Before we can assess answers to either question, we have to be clear about the specific foundational concerns and the general character of solutions proposed by the program. The broad background is provided by striking developments in 19-th century mathematics, namely the emergence of set theory, the discovery of set theoretic foundations for analysis, and the rise of modern axiomatics with a distinctive structuralist bent. These developments came to the fore in Cantor's and Dedekind's work. Some of the difficult issues connected with them were seen by Cantor, others were made explicit by Dedekind and by Kronecker (when criticizing Dedekind); and they clearly prompted Hilbert's foundational studies in the late 1890's. Dedekind and Kronecker were both deeply influenced by Dirichlet; their divergent development of algebraic number theory together with their general reflections pinpoint the central issues most clearly. Hilbert's Program was formulated only in the early twenties, but it evolved out of this earlier "problematic".

The program was to mediate between the opposing foundational views represented by Dedekind and Kronecker, and it was to address a methodological problem, to wit, the use of 'abstract' analytic means in proofs of 'concrete' number theoretic results (employed first by Dirichlet). The expressibility of parts of classical mathematics in axiomatic systems $P$ was the basic datum for conceiving of consistency proofs for $P$ as a possibly convincing approach. The basic datum, after such $P$'s had been sharpened to formal theories, allowed Hilbert to think of classical mathematics programmatically as a formula game and, thus, of the consistency problem as a syntactic one. In this way Hilbert side-stepped the philo-