1 Introduction

Although not so popular in the contemporary philosophical and logical scene, logicism dating from Frege and Russell was the first attempt to declare arithmetic as invariantly valid for any model involving an infinite number of individuals.

Now, the purpose of this paper is to locate such an invariance in a more elementary part of logic, namely, a tiny fragment of Leśniewski's ontology, and it will be shown that the fragment to be called \( L_1 \) is invariant with respect to any model including or not including individual-like names. (The said propositional fragment \( L_1 \) was introduced by Ishimoto [1977] and has subsequently been elaborated by Kobayashi-Ishimoto [1982], Inoué-Kobayashi-Ishimoto [forthcoming] and others.)

2 Tableau-method version

In this section, we shall develop the propositional fragment \( L_1 \) of Leśniewski's ontology by way of a tableau-method. As will be seen, the method will have the effect of making the proposed invariance stand out more.

Before presenting the fragment as a tableau-method system, the notion of positive and negative parts of a formula will be defined in advance although it is by no means indispensable. (The notion is essentially due to Schütte [1960, 1968]. But, we are employing its dual as developed by Schütte [1960, p. 13] and Ishimoto [1986] (cf. Inoué [to appear, a]), and by Ishimoto [1970] and Shimizu [1990] in connection with constructive logic.) Thus, a formula is a thesis in the proposed system iff its negation is provable.
**Definition 2.1** The *positive* and *negative parts* of a formula $A$ are defined only as follows:

2.11 $A$ is a positive part of $A$,
2.12 If $B \land C$ is a positive part of $A$, then $B$ and $C$ are positive parts of $A$,
2.13 If $\neg B$ is a positive part of $A$, then $B$ is a negative part of $A$,
2.14 If $\neg B$ is a negative part of $A$, then $B$ is a positive part of $A$.

The (well-formed) formulas are defined in the well-known way in terms of the atomic formulas of the form $eab$ (with $e$ being Leśniewskian epsilon), a countably infinite stock of free name variables, $a, b, c, \ldots$, two logical symbols, namely, $\land$ (conjunction) and $\neg$ (negation) as well as some auxiliary symbols. (The outermost parentheses are always suppressed.)

The (well-formed) formulas thus defined will be denoted by meta-logical variables such as $A, B, C, \ldots$. The primitive symbols and their legitimate combinations will be understood only meta-logically.

For the sake of reference, we shall hereunder present some examples of the occurrences of a formula or formulas in another as its positive or negative parts. (Essentially following Schütte, $F[A^+]$ ($G[A^-]$) is to the effect that $A$ is a positive (negative) part of $F[A^+]$ ($G[A^-]$).)

\[
\begin{align*}
F[A^+] &= A, \\
F[A^+] &= \neg \neg A, \\
F[A^+] &= \neg (\neg A \lor B) (= \neg \neg (\neg A \land \neg B)), \\
G[A^-] &= \neg A, \\
G[A^-] &= \neg \neg \neg A, \\
G[A^-] &= \neg \neg \neg (\neg B \lor A) (= \neg \neg \neg (\neg B \land \neg A)), \\
F[A^+, B^-] &= \neg \neg A \land \neg \neg B, \\
G[A^-, B^+] &= \neg (\neg A \supset \neg B) (= \neg (\neg A \land \neg B)),
\end{align*}
\]

where $A$ and $B$ are different formulas. This is necessary for securing their unique occurrences as a positive or negative part in the given formula.

Like other tableau-methods, ours is also defined by way of a number of *reduction rules* which are as follows: