INVERSE PROBLEM OF REFLECTION AND TRANSMISSION FOR A BIANISOTROPIC MEDIUM

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Abstract. On the basis of the surface impedance and characteristic matrix methods, the exact solutions of the inverse problems of reflection and transmission for motionless and uniformly moving bianisotropic media with temporal and spatial dispersion are obtained.

1. Introduction

Recently, considerable attention has been focused on the properties of chiral, biisotropic and bianisotropic media, especially, layered media and composite materials. Within the last few decades, several sets of constitutive equations for such media have been proposed, and their merits and shortcomings as well as the number of the independent material parameters and their properties have been intensively discussed in literature [1]–[8]. The progress in constructing artificial chiral media provides new impetus for theoretical studies concerning predictions of the effective media properties of composite materials and development of new techniques for measuring electromagnetic parameters of biisotropic and bianisotropic media [9]–[13].

Bianisotropic media are characterized by the constitutive equations [1, 3, 4, 5]

\[ D = \varepsilon E + \alpha B, \quad H = \beta E + \mu^{-1}B \]  

where \( \varepsilon \) and \( \mu \) are the permittivity and permeability tensors, \( \alpha \) and \( \beta \) are the pseudotensors of gyrotropy. Using the four-dimensional tensors...
\[ F = \sum_{1 \leq i < j \leq 4} F_{ij} \dot{\theta}^i \wedge \dot{\theta}^j \]
\[ = B_3 \dot{\theta}^1 \wedge \dot{\theta}^2 - B_2 \dot{\theta}^1 \wedge \dot{\theta}^3 + B_1 \dot{\theta}^2 \wedge \dot{\theta}^3 + (E_1 \dot{\theta}^1 + E_2 \dot{\theta}^2 + E_3 \dot{\theta}^3) \wedge \dot{\theta}^4 \] (2)

\[ G = \sum_{1 \leq i < j \leq 4} G_{ij} \dot{\theta}^i \wedge \dot{\theta}^j \]
\[ = H_3 \dot{\theta}^1 \wedge \dot{\theta}^2 - H_2 \dot{\theta}^1 \wedge \dot{\theta}^3 + H_1 \dot{\theta}^2 \wedge \dot{\theta}^3 + (D_1 \dot{\theta}^1 + D_2 \dot{\theta}^2 + D_3 \dot{\theta}^3) \wedge \dot{\theta}^4 \] (3)

one can replace Eqs. (1) by the Lorentz-covariant constitutive equation

\[ G = MF = \sum_{1 \leq i < j \leq 4, 1 \leq k < l \leq 4} M_{ij}^{kl} F_k \dot{\theta}^j \wedge \dot{\theta}^i \] (4)

where

\[ M = \sum_{1 \leq i < j \leq 4, 1 \leq k < l \leq 4} M_{ij}^{kl} \dot{\theta}^i \wedge \dot{\theta}^j \otimes \epsilon_k \wedge \epsilon_l \] (5)

is the four-dimensional material tensor. Here, \((e_i)\) and \((\dot{\theta}^i)\) are the dual orthonormal bases in the Minkowski vector space \(V\) and its dual \(V^*\) (the space of 1-forms or, in other words, the space of covariant vectors), \(e_i \otimes \dot{\theta}^j = e_i [\dot{\theta}^j = \delta^j_i, \ i, j = 1, 2, 3, 4, \ \delta^j_i\) is the Kronecker delta function, \(\otimes, \wedge\) and \([\) are the tensor, exterior and interior products \([11, 12]\). For instance, \(e_i \wedge e_j = e_i \otimes e_j - e_j \otimes e_i, \ e_1 [(\dot{\theta}^1 \wedge \dot{\theta}^2) = \dot{\theta}^2, (e_1 \wedge e_2)[\dot{\theta}^1 = e_2].\)

In \([11, 12]\), by making use of the exterior algebra and the generalized impedance and characteristic matrix methods, the Lorentz-covariant solutions of direct and inverse problems for dispersionless linear media are obtained. In particular, one can use these solutions to find all 36 material parameters \(M_{ij}^{kl}\) by measuring reflection and transmission coefficients.

In a homogeneous linear dispersive medium, \(G\) and \(F\) are related by the integral transformation \([5, 14]\)

\[ G(x) = \int \mathcal{M}(y) F(x - y) d^4 y \] (6)

where \(d^4 y\) is the infinitesimal element of the space-time volume, and \(\mathcal{M}\) is a tensor function of type \((2,2)\). For a harmonic partial wave, Eq. (6) reduces to Eqs. (1) and (4), but \(\varepsilon, \mu, \alpha, \beta, \) and \(M\) now depend on the wave vector \(\mathbf{k}\) as well as on the frequency \(\omega\) (the four-dimensional wave vector \(\mathbf{K} = \mathbf{k} + (\omega/c) \mathbf{e}_4\)) \([5, 14]\). Instead of using a separate set of such tensors for each partial wave, it is advantageous to extend the material equation (4) to some superpositions of waves by introducing the generalized material tensor defined on a set of evolution operators \((F(x + y) = \mathcal{F}(y)F(x))\) by \([12, 13, 15, 16]\)

\[ M(\mathcal{F}) = \int \mathcal{M}(y) \mathcal{F}(-y) d^4 y \] (7)