CONCEPTS OF LINEAR AND NONLINEAR FRACTURE MECHANICS

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ABSTRACT

After a brief introduction, various fracture mechanics concepts and fracture criteria are examined and their relative merits are discussed. There are two distinct aspects of fracture in the case of several practical materials exhibiting nonlinearity. These are, crack extension and crack growth. Sih's criterion for predicting the crack extension direction and critical fracture load can be applied for non-self-similar crack extension unlike Griffith's criterion which was based on self-similar crack extension.

Fracture toughness testing of many practical alloys indicates that most of the effects due to plasticity can be attributed to the phenomenon of slow crack growth. Several of the proposed crack growth laws are examined in this paper and it is shown that the results obtained using the plastic energy method of Lee and Liebowitz have been successfully applied to several uniaxial and biaxial loading cases.

INTRODUCTION

Linear elastic fracture mechanics has made considerable progress recently with regard to several practical crack problems. Griffith [1] had originally proposed a global energy criterion for brittle fracture based on the assumption that crack extension is self-similar. This enabled him to treat crack length as a scalar and obtain the scalar derivative of the global energy with respect to crack size. However, crack extension is non-self-similar in several practical cases requiring a different approach to the problem. Noting this Sih [2] has provided a crack extension criterion based on minimizing the crack tip strain energy density in order to obtain the direction of crack extension as well as the critical fracture load. Sih's criterion has been applied to several two-dimensional and three-dimensional crack problems where crack extension is non-self-similar [3-4]. Griffith's criterion and Sih's criterion are applicable only when the influence of crack tip plasticity is negligibly small. However, the influence of plasticity on fracture is considerable in several high strength alloys. Recent studies on crack tip plasticity and slow crack growth have provided a better understanding of fracture in the nonlinear range. Lee and Liebowitz [5] have indicated that many of the experimental observations on the influence of load parallel to the
Crack on fracture characteristics can be explained based on the nonlinear finite element analysis. For example, the critical fracture load is influenced by the load parallel to the crack, in the nonlinear range; the effect is more pronounced when there is significant compressive load parallel to the crack.

Fracture toughness testing of practical alloys indicates that most of the effects due to plasticity on fracture toughness can be attributed to slow crack growth phenomenon. Several theories have been proposed to characterize slow crack growth. If the crack growth proceeds in a self-similar manner, the crack growth law can be expressed as a scalar equation. In the case of non-self-similar crack growth, a vector equation should be used to postulate the crack growth criterion. Several criteria have been proposed for the case of self-similar crack growth of which the criterion based on constant dJ/da presented by Paris et al. [6] and the criterion based on a constant plastic energy rate dP/da proposed by Lee and Liebowitz [7] are based on the global energy. Lee and Liebowitz's method may be extended to non-self-similar crack growth. For the case of slow crack growth accompanied by crack tip unloading J appears to be path dependent as indicated by the results of Shih et al. [8]. On the other hand, Lee and Liebowitz's method [7] is based on incremental plasticity theory and hence can account for crack tip unloading during slow crack growth. Recently, Liebowitz et al. [9,10] have successfully applied the plastic energy method to several uniaxial and biaxial fracture problems. The above-mentioned aspects and the relative merits of various fracture criteria are discussed in this paper.

**LINEAR ELASTIC FRACTURE MECHANICS AND CONCEPTS OF CRACK EXTENSION**

Linear elastic fracture mechanics deals with the behavior of cracked structures using linear elasticity theory. Pioneering efforts by several researchers such as Griffith [1], Irwin [11], Williams [12], Sih [13], Sih and Liebowitz [14], Sneddon [15], Paris and Sih [16], and Cherepanov [17] enable the characterization of crack tip stress field for the case of many two- and three-dimensional crack problems. Recent results of Hartranft [8] indicate that the square root singularity of the crack tip stress field (\(\sqrt{r}\) singularity) is common to all types of external loads including extensional and bending loads. Also, the angular distribution of stresses around the crack tip depends only on the principal mode of loading, symmetric, skew symmetric, or anti-plane mode. For a general case of loading, the crack tip stress field can be expressed as shown by Sih [13] in the following form.

\[
\sigma_{rr} = r^{-1/2}\left[A_1(z)\left(\frac{1}{2}\cos(3\theta/2) - \frac{5}{2}\cos(\theta/2)\right)\right. \\
\left. + A_2(z)\left(\frac{1}{2}\sin(3\theta/2) = \frac{5}{6}\sin(\theta/2)\right)\right]
\]

\[
\sigma_{\theta\theta} = -r^{-1/2}\left[A_1(z)\left(\frac{1}{2}\cos(3\theta/2) + \frac{3}{2}\cos(\theta/2)\right) \\
\left. + A_2(z)\left(\frac{1}{2}\sin(3\theta/2) + \frac{1}{2}\sin(\theta/2)\right)\right]
\]