CHAPTER 3

General Small-Signal, Low-Frequency Analysis of Switching Regulators

3-1 Introduction

The switching cell is one of the functional blocks in the closed-loop regulator structure. With the regulating loop closed, the controlled quantity of the switching cell is no longer an independent variable; it is determined by other functional blocks in the closed-loop system. The system shown in Figure 3-1 operates as follows. The input to an error amplifier $G$ is an error signal: the difference between the output voltage $u$ and a reference level $E_{\text{REF}}$ (the desired value of the output voltage). The Laplace transform of the ac component of the output voltage is $U'$. The error-amplifier output is the driving signal for circuit $N$, whose output defines the controlled quantity of the switch.

![Figure 3-1. Small-signal, low-frequency model of a switching regulator.](image-url)
Regardless of the detailed internal structure of circuit $N$, the small-signal characteristic of $N$ can be represented as

$$N = \frac{X^*}{U_1^*} \quad (3-1)$$

In general, the small-signal characteristic of $N$ is a function of frequency and the operating point in the large-signal domain. The dimension of $N$ (e.g., volts per volt or duty ratio per volt) depends on the nature of the controlled quantity $X^*$ and the voltage $U_1^*$ (the input to block $N$).

Many different realizations of the functional block $N$ are possible because of the variety of possible controlled quantities. In general, this block is nonlinear, so it must also be linearized before the entire regulator can be analyzed. However, obtaining these linearized models is sometimes complicated. It requires a considerable analytical effort, even after making certain simplifying assumptions and approximations. Using two examples, the next section briefly demonstrates the development of a general procedure that leads to linear models of the block $N$.

### 3-2 Modulator Transfer Functions

Circuits that produce a duty ratio as a controlled quantity dependent on an input voltage are called *pulse-width modulators*, *duty-ratio modulators*, or, simply, *modulators*.

Two types are especially important: the *natural-sampling* modulator (a comparator) and the *uniform-sampling* modulator [1]. The waveforms of a comparator are represented in Figure 3-2. (The comparator is the most commonly used type of modulator.) A ramp voltage with a constant frequency $1/T$ and an amplitude $V_M$ is compared to the control voltage $u_i(t)$. An output pulse is initiated at the beginning of every ramp. The pulse is terminated when the ramp voltage exceeds the control voltage. The pulse duration is $t_c$.

Thus, an output pulse of a duration $t_{ca}$ is related to the instantaneous value $v_a$ of the control voltage by the relation

$$v_a = \frac{V_M}{T} t_{ca} \quad (3-2)$$

Similarly, for the following interval, one finds

$$v_b = \frac{V_M}{T} t_{cb} \quad (3-3)$$