VI. Two-Dimensional Analysis

Our previous emphasis to relate an optical image to material details of an object involved principally the analysis of the converter response along a linear traverse of a neutron radiograph. We next extend this one-dimensional analysis to two dimensions, again emphasizing the conceptual-mathematical basis as well as selected applications.

6.1 ONE AND TWO DIMENSIONS

One significant result in Chapters 4 and 5 is the demonstration that a line-spread function,

\[
L(x, x_0) = \frac{\lambda/\pi}{1 + \lambda^2(x - x_0)^2}
\]

(6.1)

and its various integrants

\[
\psi(x) = N \int_{-\infty}^{\infty} \phi_t(x_0) L(x, x_0) dx_0
\]

(6.2)

provide useful tools in establishing some precise object-image relationships. The interpretation of the function \(L(x, x_0)\) is that it represents the converter response on the image plane at an arbitrary \(x\)-coordinate due to an infinitesimally narrow line source of neutron radiation at \(x_0\) in the object plane.

Actually, the coordinate \(x_0\) was defined relative to the objects of interest in Chapters 4 and 5 and not relative to a conceptual object plane, conveniently defined as being orthogonal to the neutron beam and placed adjacent to the object on the exit side of the neutron beam. While this subtle difference was previously unimportant and leaves the previous analysis unaltered, we will now find it simpler to define the position of the neutron sources relative to an object plane, Fig. 1.4. Retaining the convention that the object plane Cartesian coordinates are \((u,v)\), corresponding to \((x,y)\) in the image plane, we can assert that the neutron line source is at the line \(u_0\) in the object plane rather than \(x_0\) in the object. Mathematically, this redefinition is just
The line-spread function is one-dimensional since it describes an image plane response with variation in only one of the possible directions, as graphically depicted in Fig. 6.1. Note that the x-y plane and the u-v plane are rigidly related by having their corresponding axes parallel and sharing the same z-coordinate.

\[
L(x,x_0) \rightarrow L(x,u_0) = \frac{\lambda/\pi}{1 + \lambda^2 (x - u_0)^2}
\]

Fig. 6.1: Line-spread function in two dimensions associated with a neutron line source at \(u_0\).

With the depiction of the one-dimensional line-spread function firmly established, it is a self-evident extension to depict a two-dimensional point-spread function, as suggested in Fig. 6.2. The associated point-spread function, \(P(x,y,u_0,v_0)\), is therefore the image plane response at an arbitrary point \((x,y)\) in the image plane, attributable to a point source of neutron radiation passing through an infinitesimal hole at the coordinate \((u_0,v_0)\) on the neutron absorbing planar object, or, using equivalent terminology, at \((u_0,v_0)\) in the object plane.