CHAPTER 10

COLLISIONS, PARTIAL REDISTRIBUTION, AND TURBULENT MAGNETIC FIELDS

10.1. Introduction

In the preceding chapters we have developed the foundations for treating general problems involving radiative scattering in a magnetic field. The theory however needs to be completed by including the effects of collisions and partial redistribution before it is ready for general use on the sun.

It would take us too far to make a comprehensive treatment of collisional physics here. Instead we start with a simplified classical approach to gain some physical insight about collisional effects and then treat the quantum-mechanical theory in a phenomenological way. Thereby we will take over from literature the numerical values of the two main parameters of collision theory, the rates of collisional destruction of the atomic orientation (\(\gamma_c^{(1)}\)) and alignment (\(\gamma_c^{(2)}\)), expressed in units of the total elastic collision rate \(\gamma_e\) that is responsible for line broadening.

A preliminary overview of partial redistribution (the relation between the incoming and outgoing frequencies and the scattering directions in the observer's coordinate system) in terms of the Mueller matrix formalism was given in Chapter 5 for classical scattering. This theory will now be completed and integrated with the collisional theory. The resulting redistribution matrix incorporates the results from the best available detailed quantum-mechanical calculations. Analytical expressions will be given at various levels of approximation. Particularly useful are the approximate forms in the line core and wings, respectively. In the general radiative transfer equation (e.g. Eq. (6.104)) all the physics of the coherence phenomena, collisions, and partial frequency redistribution resides in the redistribution matrix \(R\).

After these theoretical tools have been established we turn to a discussion of practical applications of the Hanle effect, in particular for the diagnostics of the magnetic field structure in prominences, in the chromosphere and above, and of turbulent magnetic fields at various levels of the atmosphere. Since the elusive turbulent fields are of fundamental importance for our understanding of solar magnetism, we will describe how they may be constrained in different ways by observations of Hanle depolarization, line broadening, and the transverse Zeeman effect, although they remain spatially unresolved. These joint constraints provide information on both the turbulent field strength and the angular distribution of the field vectors.
10.2. Classical Collision Theory

The spectral properties of a fluctuating component $E_q(t)$ of the radiation field is found by forming the Fourier transform $\tilde{E}_q(\omega)$ and the power spectrum $\tilde{E}_q\tilde{E}_q^*$. Thus we may Fourier transform the Jones vector $J$ of Eq. (2.28) to get $\tilde{J}$. Since in the Jones calculus all polarizing filters are represented by linear operators (cf. Eq. (2.29)), and since the Fourier transform is itself a linear operator, the whole Jones calculus remains valid if we replace all electric fields with their Fourier transforms. We may then form a coherency matrix $D_{FT} = \tilde{J}\tilde{J}^\dagger$ from the Fourier transforms as in Eq. (2.33) as well as a Stokes vector $S_{k,FT} = \text{Tr} (\sigma_k D_{FT})$ as in Eq. (2.43). The whole Mueller calculus then applies to $S_{k,FT}$. In particular $S_{0,FT}$ is the power spectrum that is recorded with spectrographs and detectors. The spectral behavior of the polarization effects (including the Hanle effect) is therefore contained in the bilinear products formed by $\tilde{J}\tilde{J}^\dagger$. In terms of the spherical vector components we have to explore the products $\tilde{E}_q\tilde{E}_q^*$.

As we have seen in Sects. 3.2 and 3.12, the spherical vector component $E_q$ of the electric field of the scattered radiation is proportional to the corresponding component $d_q$ of the electric dipole moment of a classical oscillator. It is thus sufficient to discuss the properties of $d_q$ and regard them as representative of $E_q$.

A classical oscillator that has been excited will perform damped oscillations governed by Eq. (3.33) without the driving term on the right-hand side after the exciting radiation has been turned off (the photon absorption has taken place). The solution of this equation gives the time dependence of the oscillating electric dipole moment. To first order in the classical damping constant $\gamma_N$ of Eq. (3.33) and the Larmor precession frequency $\omega_L$ of Eq. (3.44), the damped oscillations of the spherical vector component $d_q$ of the electric dipole moment are described by

$$d_q = d_{0q} e^{i(\omega_0 + q\omega_L)t} e^{-((\gamma_N/2)t)}.$$  \hspace*{0.5cm} (10.1)

These oscillations can be viewed as an antenna that generates the emitted (scattered) radiation.

To get the spectral properties of the scattered radiation we need to calculate the Fourier transform $\tilde{d}_q(\omega)$ of $d_q(t)$. The Fourier integral however cannot extend over infinite time, since the damped oscillation of Eq. (10.1) only lasts as long as the excited atom is undisturbed by collisions. Accordingly it only exists between time $t = 0$ (time of excitation) and $t = t_c$ (time of a collision, which is assumed to instantly scramble the phase of the oscillation). In the presence of collisions $\tilde{d}_q$ is therefore given by

$$\tilde{d}_q \sim \int_0^{t_c} e^{i(\omega_0 - \omega + q\omega_L + i\gamma_N/2)t} \, dt = \frac{i [1 - e^{i(\omega_0 - \omega + q\omega_L)t_c} e^{-(\gamma_N/2)t_c}]}{\omega_0 - \omega + q\omega_L + i\gamma_N/2}. \hspace*{0.5cm} (10.2)$$

The spectral properties of the coherency matrix and the Stokes parameters depend on the bilinear products $d_q^* d_q$. Let $\tau_c$ be the average time between two collisions, which are assumed to occur randomly according to Poisson statistics.