CHAPTER 5

CLASSICAL SCATTERING
AND THE HANLE EFFECT

5.1. Coherent Scattering in the Rest Frame

Derivation of the scattering emission vector to be used in the Stokes transfer equation requires integration over all the angles of incidence, which implies that we make a superposition of many scattering events. As the different scattering events are stochastically independent of each other, without any preferred phase relations between them, the superposition is incoherent. Still this incoherent superposition applies to the treatment of coherent scattering, since the coherence is between the three oscillating dipole components (in the classical description) within each single scattering event. This coherence is fully described by the Jones matrix for a single scattering process, which was derived in Chapter 3 and given by Eqs. (3.84), (3.85), and (3.88).

The incoherent superposition of many scattering events cannot be done within the Jones formalism, but requires use of the coherency matrix or the Stokes formalisms. For our discussion of scattering, we will stay within the Stokes formalism. The Mueller matrix $M_{sc}$ for a single scattering process can be directly derived from the corresponding Jones matrix, which according to Eq. (3.84) is

$$
\frac{\omega^2}{2\pi c^2}\frac{e^{ikr}}{r} w.
$$

Using Eqs. (2.39) and (2.48) we then obtain

$$
M_{sc} = \frac{\omega^4}{4\pi^2 c^4} \frac{1}{r^2} T (w \otimes w^\ast) T^{-1},
$$

where the transformation matrices $T$ and $T^{-1}$ are given by (2.49).

If the incident Stokes vector (per unit solid angle) is $I'$, propagating in a direction characterized by colatitude $\theta'$ and azimuth $\phi'$, then the Stokes vector per unit solid angle, scattered in the direction defined by $\theta$ and $\phi$ and denoted by the vector $j_{sc}$, is obtained by integrating over all the incident directions, thus adding up the stochastically independent contributions from all the incident solid angles $d\Omega'$:

$$
j_{sc} = \int r^2 M_{sc} I' \ d\Omega'.
$$

The $r^2$ factor compensates for the $1/r^2$ factor in $M_{sc}$, such that $r^2 M_{sc}$ represents scattering from one unit solid angle into another unit solid angle.
Inserting Eq. (5.2) into (5.3), we thus obtain

\[ j_{\text{sc}} = \frac{\omega^4}{\pi c^4} \int T(w \otimes w^*) T^{-1} I' \frac{d\Omega'}{4\pi}. \]  

(5.4)

\( w \), given by Eq. (3.85), is a sum over \( q \) (index for the three spherical vector components) containing the magnetic-field dependent dispersion factor \( n_q - 1 \) and the two \( q \)-dependent \( \varepsilon \) factors describing the geometry of the incident and scattered rays. The tensor product \( w \otimes w^* \) in Eq. (5.4), explicitly given by Eq. (2.39), therefore contains not only the squared terms, but also the cross products connecting different \( q \) values with each other. It is these terms that represent the interference or coherence effects, and the magnitude of these effects depends on the strength and direction of the magnetic field. The magnetic-field dependent polarization phenomena resulting from this interference are commonly called the Hanle effect.

### 5.2. Frequency Redistribution of Polarized Radiation

Eq. (5.4) is valid in the rest frame of the scattering atom, in the absence of collisions or Doppler shifts. In scattering problems (as opposed to pure absorption problems) the effect of Doppler motions is not merely a matter of line broadening but of redistribution, since scattering is a two-photon process with the frequency shift being coupled to the angular redistribution. Polarization adds a new dimension of complexity, since the polarization properties are directly linked to the angular redistribution and are thereby, via the Doppler motions, coupled to the frequency redistribution.

In contrast to the scalar redistribution function used in current radiative transfer literature, the treatment of polarization problems requires the introduction of a \( 4 \times 4 \) redistribution matrix \( \mathbf{R}(\nu, \mathbf{n}; \nu', \mathbf{n}') \), defined as follows: If the incident Stokes vector is \( I_\nu'(\nu', \mathbf{n}') \), propagating in direction \( \mathbf{n}' \) at frequency \( \nu' \), then the probable contribution to the Stokes vector scattered in direction \( \mathbf{n} \) at frequency \( \nu \) from differential solid angle \( d\Omega' \) and frequency interval \( d\nu' \) is

\[ \sigma \mathbf{R}(\nu, \mathbf{n}; \nu', \mathbf{n}') I_\nu'(\nu', \mathbf{n}') \frac{d\Omega'}{4\pi} d\nu'. \]  

(5.5)

\( \sigma \) is the scattering coefficient, the magnitude of which follows from the above definition and the normalization condition for \( \mathbf{R} \):

\[ \int \frac{d\Omega'}{4\pi} \int \frac{d\Omega}{4\pi} \int d\nu' \int d\nu \mathbf{R}_{11} = 1, \]  

(5.6)

where \( \mathbf{R}_{11} \) is the matrix component in the first row and column of \( \mathbf{R} \). It can be expressed as the vector product

\[ \mathbf{R}_{11} = \mathbf{1}^T \mathbf{R} \mathbf{1}, \]  

(5.7)

where \( \mathbf{1} \) is the unit four-vector (introduced in Eq. (4.39)), and \( \mathbf{1}^T \) is its transpose. This type of product will be of later use below.