CHAPTER X

A TALE OF FINE STRUCTURE COINCIDENCES

1. Summary

Here is an odyssey in the realm of spectral secrets of Nature's simplest atom. A modification of Weber's force function, recently proposed by Phipps as an alternative to the Coulomb potential gives a fine structure of twice the observed amount. Yet the two relativistic approaches by Sommerfeld and Dirac, with or without this Weber-Phipps modification, both give the observed fine structure splitting, except for higher order differences in even powers of $\alpha$. Pauli's approximation, however, in conjunction with the electron's magnetic moment anomaly, gives, by far, the best agreement with experimental observation. The major reason may well be that it permits, in addition to a correction for the comoving nucleus, also odd powers of $\alpha$ such as introduced by the accurately known magnetic moment anomaly of the electron. An inspection of the traditional QED calculations of the Lamb shift reveals for s-states a residual distance of effective proximity between electron and nucleus. Model-based interpretations of this residual distance leave many open questions.

2. Introduction

One would think that a particle moving in a central field might be one of the most exhaustively treated topics in the physics literature. In taking this conclusion for granted, one would therefore hardly expect modifications that could contribute much in terms of deeper physical insight. Nevertheless, even this early and great breakthrough of modern science is by no means a closed chapter in the contemporary annals of physics. It has retained a multitude of questions that today are of as much interest as in the days of Newton.

In the course of time, the extension from one- and two-body situations to three- and more body situations have presented very serious hurdles in obtaining closed-form solutions that could give an immediate insight in the stability of such compound structures. Over and above, the formidable mathematical obstacles presented by more bodies carried over into the modern realms of relativity and quantum theory. Hence, notwithstanding these latter day revolutions, some problems have not changed.
After Newton's fundamental solution for the case of the ideal inverse square law, the most commonly considered variations of the classic planetary problem have to do with changes in the force law, say due to deviations in the ideal inverse square law, changes due to proximity effects or perturbations due to other gravitating bodies, and, last but not least, interactions with rigid body dynamics and continuum mechanics due to the finite size of the interacting objects. In dealing with these matters, Euler, Lagrange, Hamilton and Jacobi have given Newtonian theory its mathematically and physically most-sophisticated form, a form which subsequently was found to be almost ideally suited for accommodating the subsequent developments of relativity and quantum theories.

The great Newtonian breakthrough in mechanics ironically was made possible by the introduction of a hybrid conceptual feature. The mixed use of field and particle notions provided a major key to solving problems in celestial mechanics. The kinematic aspects of the theory are particle-based, yet the force interaction between particles or bodies is based on the use of what has become known as a field concept. This situation prevails for gravitational as well as for electrical interactions of the Coulomb type.

In one of his later publications, Brillouin¹ called attention to this dichotomy associated with the notions of kinetic and potential energy. While the kinetic energy of a particle can be clearly associated with an isolated particle, the potential energy, by contrast, is always a mutual affair of the particle and objects creating a "field" at the location of the particle. Yet Newtonian treatments have, in the course of time, led to a tacit convention in which kinetic and potential energy are uniquely related to one and the same particle. Kinetic energy relates to a given inertial frame, and could be said to be defined with respect to the rest of the universe.

When the field concept first was introduced by Newton for dealing with his universal law of gravitation, it was the first mathematical coding of what then was called an "action at a distance," as distinguished from the more familiar contact forces. Newton concluded this action at a distance to be a fact of nature, without extensive considerations as to how this action at a distance might come about physically. Except, perhaps, for an added presumption of instantaneous interaction between two physically separated locations in space, there were no further specifications as to what might be taking place in the space between the objects. In the course of time, Coulomb and Ampère were destined to add new examples testifying to nature's ability to act over distance without the need for an explicitly observable physical contact.

When the development of Maxwell theory taught us to identify the Coulomb and Ampère forces as near-field manifestations of a general, time-delayed, radiation field, it was bound to affect the earlier thesis of action at a distance. For the Coulomb and Ampère forces, at least, new