On May 29, 1883, Corrado Segre took his doctorate in Turin (Torino), under Enrico D'Ovidio’s guidance. His thesis (Segre 1884a,b) was published one year later in the Journal of the local Academy of Science, and after a short time it became a fundamental starting point for the development of Italian projective n-dimensional geometry.

The paper is devoted to the general study, by geometrical tools, of n-dimensional vector spaces (projective spaces), and in particular of bilinear forms defined on them (quadrics). At the very beginning of the paper, Segre gratefully acknowledges Grassmann’s role as his forerunner. It is very likely that Segre did not utilize many of Grassmann’s ideas in this first work, but he had obviously perceived their importance in providing sounder foundations to the new geometrical edifice.

Indeed Segre writes:

“We must note that [...] Grassmann, in his Ausdehnungslehre, has already developed, together with many other important ideas, that of n-dimensional geometry in 1844, but his work has been taken up only recently. The following very important passage is found in one of Grassmann’s papers of 1845 [...] and it shows that the Ausdehnungslehre in the mind of this inventive scholar was indeed what we now name «n-dimensional geometry»: Meine Ausdehnungslehre bildet die abstrakte Grundlage der Raumlehre (Geometrie), d. h. sie ist die von allen räumlichen Anschauungen gelöste, rein mathematische Wissenschaft, deren specielle Anwendung auf den Raum die Raumlehre ist ... die Raumlehre, die auf etwas in der Natur gegebenes, nämlich den Raum, zurückgeht, ist kein Zweig der reinen Mathematik, sondern eine Anwendung derselben auf die Natur” (Segre 1884a, p. 26).

Since the paper in which Segre included this quotation belongs to the founding contributions to the Italian school of n-dimensional geometry, I consider it useful to clarify some points, if only briefly. The work of the Italian School, and in particular that of C. Segre and E. Bertini, rests on four correlated pillars: Plücker’s so-called line-geometry, which was interpreted as a starting point towards a new idea of multi-dimensional space in which the “points” were able to assume different meanings; Grassmann’s point of view with regard to “abstract” linear space as the natural framework for any particular geometrical interpretation; von
Staudt’s notion of the “coordinatization” of synthetic projective space as a fundamental tool for the systematic translation of geometrical concepts into algebraic ones (and vice versa); and finally, algebraic ideas presented by Weierstrass and Frobenius, but suitably translated into geometrical terms.

To be more specific, whereas Plücker suggested to Segre the basic geometrical intuitions, Grassmann allows him to place these same ideas within a much sounder conceptual framework, a broader philosophical view, a more acute formalism. Grassmann’s work indeed clearly suggests that the natural environment where we can study multi-dimensional projective geometry is what we now call an abstract vector space (over the complex field).

A closer scrutiny of this will show how the young Segre’s ideas on linear spaces, which in the beginning were rather confused, became - also owing to a more and more careful study of the Stettin scholar’s work - clearer and ever more precise, and how they finally gave rise, in particular through the contributions of Segre’s students, to some fundamental works on modern axiomatics in geometry.

First, a brief sketch on how the first ideas of multidimensional space emerged. It goes back to the beginning of the 19th century and was at first closely tied to mere analytical fact, just to give a geometrical aspect to the use of objects defined through $n$-numerical parameters. During the next decades, this idea grew into a clearer and more geometrical one.

The 1840s saw publication of several papers in which the idea of multidimensional space appears clearly stated. Besides Grassmann’s *Ausdehnungslehre*, there are the papers (Cayley 1845), (Hamilton 1844) and in particular (Plücker 1846).

It was indeed Plücker’s approach which had the greatest impact. In his influential paper (Plücker 1865), and above all in his treatise (Plücker 1868-69), he considered as a “point” in a $n$-dimensional space an arbitrary object from ordinary (tri-dimensional) space depending on $n$ parameters. Thus, a line may be written as

\[
\begin{align*}
x &= rz + a \\
y &= sz + b
\end{align*}
\]
determining a point $(r, a, s, b)$ in four-dimensional space. The set of points satisfying a given algebraic equation is called a complex (a hypersurface in four-dimensional space), the intersection of two complexes a congruence and the intersection of three complexes a configuration.

In line with Plücker’s approach, many mathematicians extended his results during the second half of 19th century. In this trend, Felix Klein (considering the minors of the system of equations which determine a