1: Accuracy and errors

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Tests give rise to numerical results for properties such as modulus, flow stress etc. but quoting a result without an estimate of its accuracy is only of limited use. For example a specimen may be measured and its length quoted as 10 mm. Conventionally this may be taken to mean that the length falls between 9 and 11 mm i.e. the specimen length is $10 \pm 1$ mm. However the measurement may have been taken to either greater or lesser accuracy than convention suggests. It could have been measured to 0.1 mm giving a result $10.0 \pm 0.1$ mm or alternatively taken very roughly as ‘about 10 mm’ meaning anything between 8 and 12 mm. If the accuracy is not quoted a user of the measurement is unaware of the measurement accuracy and can only guess that the measurement has been made to an accuracy of $\pm 1$ in the last figure. It is common practice to call the accuracy estimate ‘the error’ and although this may carry the implications of ‘a mistake’ the terminology is commonly used and will be used here.

SYSTEMATIC AND RANDOM ERRORS

Errors may conveniently be classified into two groups, systematic and random. Random errors and an outline of their treatment will be discussed below. Systematic errors, which are harder to deal with, are discussed first. A systematic error is often due to inaccuracy or incorrect operation of an instrument and will usually not be discovered unless a calibration of the instrument is carried out or an operator fully conversant with the instruments operation re-measures a sample. However systematic errors can arise from a wide variety of other sources. For example a series of measurements that depend on viscosity which are made on a Monday morning while the laboratory is heating up after a weekend shut down will lead to inaccuracies because viscosity falls rapidly with temperature. The slow uptake of water by a sample of nylon will lead to changes in mechanical properties which may be wrongly attributed to other reasons if the moisture content is not monitored. The use of an incorrect theory to derive a result from a set of measurements can also be considered to be a systematic error. Humans are often biased in their reading of a scale and will frequently ‘round’ to the nearest graticule mark even if accurate between mark estimates can be made. They are also prone to bias reading in the direction they wish them to go, and may tend to underestimate values if they feel that the results are coming out higher that is desired or expected or visa versa.

It is very difficult to be sure that systematic errors have been eliminated in a set of measurements since by their very nature one is often not aware of their presence. The chance of a systematic error arising can however be considerably reduced by frequent calibration of equipment, careful design of experiments, and conscious effort to be unbiased when taking readings. The use of a control experiment where the same quantity
is measured using alternative equipment or another operator also greatly assists in the elimination of systematic errors.

**RANDOM ERRORS**

Suppose a specimen of true length $x_0$ is measured by a number of experimenters and they obtain values $x_1$, $x_2$, $x_3$ etc. for the length of the specimen. It is assumed that the measurements $x_1$, $x_2$, $x_3$ etc. will be randomly distributed about the true value $x_0$ with a distribution which peaks at $x_0$. The distribution of the measurements about the true value is assumed to follow the Gaussian distribution, this is also called the Normal distribution, and it is frequently met in practice. The value quoted is the mean of the values $x_1$, $x_2$, $x_3$ etc. and the error is taken to be the standard deviation $\sigma$ of the distribution of measurements. The same is true if a single operator makes repeated measurements on one specimen and by extension if a single operator makes many measurements on a number of samples drawn from what is nominally an identical batch. The latter case may, in fact, give rise to a distribution which is not Gaussian either due to a bias on one side or other of the true mean in the samples selected, this can be eliminated by choosing a greater number of samples. However, it must be borne in mind that it is possible that the distribution of lengths is not in fact Gaussian. Non-Gaussian distributions are quite common but from the point of view of getting a value of some quantity together with an estimate of the accuracy of this value the Gaussian assumption is normally a reasonable one.

Quoting a value to $\pm \sigma$ means that there is a 68% chance that the true value will lie in the range quoted, quoting to $\pm 2\sigma$ means a 95% chance and to $\pm 3\sigma$ a 99.7% chance. It is usual to quote $\pm \sigma$ but $2\sigma$ is sometimes used and it is important that if anything other than $\pm \sigma$ is quoted that this is made clear. Most scientific pocket calculators now have inbuilt programmes to enable the mean and standard deviation of a series of measurements to be calculated.

**COMBINATIONS OF ERRORS**

It is frequently the case that a quantity is determined from a formula relating it to other more easily measurable quantities. A simple example would be the density of a cylindrical specimen. This could be obtained by weighing and measuring its length and radius then calculating the density from the expression $\rho = \frac{M}{\pi r^2 l}$. In this case, the errors in the measured quantities radius $r$, length $l$ and mass $M$ must be combined to yield the error in density. Rules to determine the errors in a derived quantity $q$ where the measured quantities are $a$, $b$, $c$ etc. and the errors in these quantities are $\Delta a$, $\Delta b$, $\Delta c$ etc. are:

For sums and differences: $q = a + b + c$... add the squares