CHAPTER 7

FINITE ELEMENT SOLUTIONS

7.1 INTRODUCTION

This book is mainly concerned with analytical and semi-analytical solutions for cavity expansion problems. In practice, however, it is inevitable that numerical methods must be used and this is particularly true when more sophisticated soil models are employed. For this reason, this chapter presents the basic formulations for finite element analysis of cavity expansion problems. To validate the finite element formulations, the analytical solutions presented in previous chapters need to be used for comparison.

7.2 UNCOUPLED DRAINED AND UNDRAINED ANALYSIS

For many practical problems, the analysis of soil behaviour can be adequately reduced to either a fully drained or an undrained problem. In this case, the uncoupled finite element formulation can be used in the calculation. In particular, the fully undrained problem can be analysed using a total stress formulation with simple plasticity models such as the Tresca and Von Mises models. On the other hand, frictional soils under fully drained conditions may be modelled by ignoring the excess pore water pressures. Finite element work using this type of uncoupled formulations for fully drained and undrained cavity expansion problems has been carried out by Carter and Yeung (1985), Reed (1986), Yeung and Carter (1989), Yu (1990), Yu and Houlsby (1990), Yu (1994a, 1996), and Shuttle and Jefferies (1998) among others.

7.2.1 Finite element formulation

The finite element formulation presented in this section was developed by Yu (1990) and Yu and Houlsby (1990) to accurately model soil behaviour.

To carry out the analysis for both cylindrical and spherical cavities at the same time, the symbol $k = 1$ for cylindrical and $k = 2$ for spherical is used. The strain rate vector $\dot{\varepsilon}$ can be expressed in terms of the radial displacement (or velocity) $\dot{u}$:

$$\dot{\varepsilon} = L\dot{u}$$

(7.1)

where

$$\dot{\varepsilon} = [\dot{\varepsilon}_r, \dot{\varepsilon}_z, \dot{\varepsilon}_\theta]^T$$

(7.2)
The stress rate vector containing the radial, axial and tangential stresses is defined as follows:

\[ \dot{\sigma} = [\dot{\sigma}_r, \dot{\sigma}_z, \dot{\sigma}_\theta]^T \]  \hspace{1cm} (7.4)

In the study of Yu (1990) and Yu and Houlsby (1990), a two-noded one-dimensional element was used so that the velocity field around a cavity is represented by values at the connecting nodes, Figure 7.1. For an element connected by nodes \( i \) and \( j \), we have the following node velocity vector:

\[ \dot{\mathbf{u}} = [\dot{u}_i, \dot{u}_j]^T \]  \hspace{1cm} (7.5)

The velocity value at any point within the element \( ij \) can be approximated by the following equation through a shape function matrix:

\[ \dot{u} = N \dot{\mathbf{u}} \]  \hspace{1cm} (7.6)

where the shape function matrix is related to the shape function associated with node number in the form:

\[ N = [N_i, N_j] \]  \hspace{1cm} (7.7)

Combining equations (7.1) and (7.6) results in:

\[ \dot{\mathbf{e}} = LN \dot{\mathbf{u}} = B \dot{\mathbf{u}} \]  \hspace{1cm} (7.8)

where the matrix linking the node velocity vector and strain vector is given by: