DYNAMICS OF COHERENT VORTICES IN LARGE-EDDY SIMULATION.

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Abstract  A fully conservative LES formulation for compressible flows, easy to implement in industrial codes, is applied with a variable-density extension of the filtered and selective structure-function models presented in Lesieur & Métais (1996, Ann. Rev. Fluid Mech., 28, 45-82). Quasi-incompressible transitional boundary layers show the establishment of a streak system of spanwise spacing ~ 100w.u. well upstream of the peak of skin friction, with some light shed on vortex dynamics thanks to the Q criterion proposed by Hunt et al. (1988, CTR-S88). Besides, a supersonic compression-ramp flow was found to develop Dean-Görtler vortices causing intense and quasi-steady spanwise fluctuations of wall heat flux.

Keywords: Large-eddy simulations, compressible flows, boundary layers, supersonic ramp flows, coherent structures, hairpin vortices, Görtler vortices.

1. Introduction

For compressible flows, several formulations of the governing equations exist, and their LES closures can yield different expressions and combinations of subgrid-scale terms. An attempt of clarification is proposed in the first section, with particular attention paid to the conservative form proposed by Normand and Lesieur (1992), which is believed to be more suitable for future high-Mach number applications in complex geometries than other non-conservative forms. Two applications are then presented, with the same numerical methods as in Normand and Lesieur (1992) and Ragab et al. (1992), viz., centered explicit McCormack-type conservative scheme, 4th-order accurate in space, and 2nd order in time (Gottlieb and Turkel, 1976). First, a well resolved LES of a spatially-growing transitional boundary layer at Mach 0.3, with emphasis on the formation of the streaks. Second, a more applied LES of
a supersonic isothermal ramp-flow, showing Görtler vortices originating from the mean streamline curvature over the hinge of the ramp.

2. Conservative Filtering Formulations

Since Leonard (1974) it has been agreed that Large-Eddy Simulations imply the resolution of the spatially filtered Navier-Stokes equations, which are conservative in nature since they result from the application of principles of conservation of mass, momentum and total (i.e. internal + kinetic) energy per unit volume, hereafter denoted $\rho E$. In the absence of genuinely multidimensional variational formulations, it is believed that it is preferable to respect this conservative nature by solving the equations in their conservation (i.e. divergence) form. To the best of our knowledge, the first compressible LES in fully conservative formulations reported in literature are those of Lesieur et al. (1991) and Ragab and Sheen (1991), further detailed in Normand and Lesieur (1992) and Ragab et al. (1992).

Assuming that the filter and the derivatives commute, the conservative filtered equations read, in co-ordinate-free notation,

$$\frac{\partial \rho}{\partial t} + \text{div} (\rho u) = 0 \quad (1a)$$

$$\frac{\partial \rho u}{\partial t} + \text{div} (\rho u \otimes u + \rho l - \sigma) = 0 \quad (1b)$$

$$\frac{\partial \rho E}{\partial t} + \text{div} [(\rho E + p)u + q - \sigma : u] = 0 \quad , \quad (1c)$$

in which $I$ denotes the unit tensor and $\otimes$ the tensor product. The molecular diffusive terms are given by the Newton and Fourier laws $\sigma = 2\mu(T)S_0$ and $q = -k(T) \text{grad} T$, respectively, in which $S_0 = \frac{1}{2} (\text{grad} u + t\text{grad} u) - \frac{1}{3} \text{div} u$ is the deviatoric (trace-free) part of the strain-rate tensor, with notation $t\text{grad} u$ for the transpose of the velocity-gradient tensor. $\mu$ and $k$ denote the molecular viscosity and conductivity, respectively. The filtered ideal (i.e. calorically perfect) gas equations of state

$$\bar{p} = R \bar{\rho} \bar{T} \quad , \quad \bar{\rho}E = C_v \bar{\rho} \bar{T} + \frac{1}{2} \bar{\rho}u : u = \bar{p}/(\gamma - 1) + \frac{1}{\gamma} \bar{\rho}u : u \quad , \quad (2)$$

are correct, together with the Stokes hypothesis made in the Newton law above, up to about 600K in air, with $\gamma = C_p/C_v = 1.4$. Beyond, $\gamma$ increases due to the excitation of the vibrations of the polyatomic molecules (see e.g. Smits & Dussauge, 1996). They remain however