Chapter 8

ON MINIMAL EVENT AND CONCRETE DATA STRUCTURES

Felipe Bracho
IIMAS, Universidad Nacional Autónoma de México
01000 México, D.F., México
bracho@servidor.unam.mx

Manfred Droste and Ingmar Meinecke*
Institut für Algebra, Technische Universität Dresden
D-01062 Dresden, Germany
{droste,meinecke}@math.tu-dresden.de

Abstract We consider concrete domains and event domains as well as their generating structures, i.e., concrete data structures and event structures. Using certain equivalence relations on the prime intervals of the domains we can associate different event structures and cds to a given domain, respectively. By introducing reductions we show the existence of a greatest event structure and cds as well as of minimal event structures and cds generating a given event and concrete domain, respectively.

Keywords: Event structure, concrete data structure, event domain, concrete domain, reduction, minimization

1. Introduction

In the theory of denotational semantics of programming languages, various kinds of systems of information and associated partial orders (domains) of information have been studied. Scott [Sco82] considered information systems and domains. Concrete data structures and concrete

*Supported by Deutsche Forschungsgemeinschaft (German Research Foundation)

domains were introduced by Kahn and Plotkin [KP78, KP93] in order to allow a fairly general semantical definition of sequentiality, see also Berry [Ber78] and Berry and Curien [BC82]. Winskel [Win81, Win87] investigated a generalization, the event structures and event domains. Event structures also bear a close relationship (categorical adjunctions) with models for concurrency like Petri nets, Mazurkiewicz traces or automata with concurrency relations, see [NPW81], [DS02] and [WN95] for a survey. A variety of further results can be found in [Cur93].

In this paper we show that given an event domain \((D, \leq)\), with respect to the present notion of reductions there always exists a greatest trim event structure \(E_{\text{max}}\) generating \((D, \leq)\), and any trim event structure generating \((D, \leq)\) can be reduced to a minimal one \(E_{\text{min}}\) also generating \((D, \leq)\). The analogous results are proved for concrete domains and trim cds.

An event structure \(E\) consists of a set \(E\) of events together with a consistency predicate for finite subsets of \(E\) and an enabling relation between consistent subsets and elements of \(E\) satisfying certain natural axioms. In the more restricted notion of a concrete data structure or cds \(M\) each event is linked to a cell, which may be filled by a value. A state or configuration of events is a subset \(X\) of \(E\) such that each finite subset of \(X\) is consistent, and each element of \(X\) can be deduced, starting from the empty set, through finitely many successive applications of the enabling relation for finite subsets and new elements of \(X\). The set of all such states of \(E\) and \(M\), respectively, partially ordered by inclusion, is denoted by \((D(E), \subseteq)\) and \((D(M), \subseteq)\). Droste [Dro89] gave with the notion of an event domain an order-theoretic characterization of all the partial orders \((D(E), \subseteq)\) where \(E\) is an event structure. This generalizes results of Winskel [Win81, Win87] who obtained the corresponding representation theorems under the additional assumption that either \(E\) is stable or that the consistency relation is induced by a binary relation of conflict on \(E\). The characterization of the partial orders \((D(M), \subseteq)\), where \(M\) is a cds, was given by Kahn and Plotkin [KP78]. They denoted these partial orders as concrete domains.

As is well-known, a given event or concrete domain may be generated by many non-isomorphic structures. This indicates that we should endow the domains with more structure in order to undo the loss of information. Indeed, as we will show here, \(E\) and \(M\) naturally induce an equivalence relation on the prime intervals of \((D(E), \subseteq)\) and \((D(M), \subseteq)\). Using these equivalence relations, called congruences, we give constructions for associating event structures or cds to a given event or concrete domain, respectively. These associated structures do not contain superfluous events; therefore we call such structures trim.