CHAPTER II

THE PSYCHOLOGICAL INTERPRETATION OF MATHEMATICAL REASONING

8. J. Stuart Mill

To show Mill's radical empiricism, I shall first of all quote his discussion of the principle of contradiction, which provides a good example of his method.¹

"An affirmative assertion and its negative are not two independent assertions, connected with each other only as mutually incompatible. That if the negative be true, the affirmative must be false, really is a mere identical proposition; for the negative proposition asserts nothing but the falsity of the affirmative, and has no other sense or meaning whatever. The Principium Contradictionis should therefore put off the ambitious phraseology which gives it the air of a fundamental antithesis pervading nature, and should be enunciated in the simpler form, that the same proposition cannot at the same time be false and true. But I can go no farther with the Nominalists; for I cannot look upon this last as a merely verbal proposition. I consider it to be, like other axioms, one of our first and most familiar generalisations from experience. The original foundation of it I take to be, that Belief and Disbelief are two different mental states, excluding one another. This we know by the simplest observation of our own minds."

There is a curious absence of logical reflection in this exposition. At first it seems that belief and disbelief are conceived as given mental states in so far as they are phenomena independently of each other. So it would be natural to interpret negation and affirmation as expressing respectively belief and disbelief. But this interpretation implies that affirmation and negation are two independent assertions linked by the fact (established by generalising certain data of introspection) that they are incompatible, and this interpretation is rejected by Mill.

In consequence we can only conceive belief according to Mill as a mental state given as a phenomenon, and disbelief as the absence of belief. However, this interpretation brings in the concept of negation, and if this concept is assumed it is no longer necessary to have recourse to observation to take account of the incompatibility of belief and disbelief.

¹ Mill, 1843, Book II, Ch. VII, Section 5.
We may also object that the very idea of an introspection which demonstrates this incompatibility gives proof of an inadequate psychology. To observe the incompatibility we should have to experience both at once, which is impossible according to Mill. But it seems to me that Mill's concept is contradicted by the facts. It may happen that we experience both at once; in such a situation we doubtless experience an internal conflict. Perhaps it is the necessity of such a conflict that Mill wishes to express when he says that belief and disbelief are mutually exclusive, but then the mutual exclusion of belief and disbelief ceases to correspond with the mutual exclusion of affirmation and negation.

In his conception of the syllogism and especially in his doctrine that every inference proceeds from particular to particular, Mill in short takes up Descartes' position as we have discussed it in Chapter I. We may say, that to a certain extent his ideas are confirmed by the analysis of logical reasoning which will be given in Section 23. However, Mill misunderstands the rôle of generalisation which is shown in particular by his attempts to reduce all general principles, even those of pure mathematics, to generalisations of certain data of experience.

9. W. Stanley Jevons' critique

Jevons tells us that he studied Mill's books more or less continuously for twenty years and that for fourteen years he had to teach from them in his university courses. It was only at the end of ten years that he discovered the fundamental falsity of what he describes as "a profoundly illogical body of writings".2

Jevons therefore decided to submit Mill's ideas to a systematic examination.²

"But, for my part, I will no longer consent to live silently under the incubus of bad logic and bad philosophy which Mill's Works have laid upon us... If, as I am certain, Mill's philosophy is sophistical and false, it must be an indispensable service to truth to show that it is so. This weighty task I at length feel bound to undertake."

Jevons' premature death prevented him from carrying out his programme. He was still able to publish, from 1877 to 1879, four critical articles on different aspects of Mill's philosophy. These articles are reproduced in Pure Logic, a posthumous collection, with a fifth study which

² Jevons, 1890, p. 201.