Problem 50: Fatigue Crack Growth and Residual Strength in a Simple MSD Problem

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1. Problem

A major problem for the structural integrity of aging aircraft components is the multiple site damage (MSD) phenomenon. MSD usually refers to multiple cracks emanating from adjacent rivet holes that can reduce the fatigue strength of the component. As a simple example consider the problem depicted in Fig. 1. To simplify the solution we can consider that the two cracks (and holes) do not interact and that the plate is wide enough to avoid edge interaction. The plate is subjected to constant amplitude fatigue loading from 0 to 130 MPa. The initial crack lengths are \( a_1 = 6 \text{ mm} \) and \( a_2 = 3 \text{ mm} \), the radius is 3 mm and the width of the plate is 80 mm. Evaluate the failure mode and the number of cycles to failure using an incremental crack growth approach. The material behaviour obeys the following rule:

\[
\frac{da}{dN} = 2 \times 10^{-12} (\Delta K)^4
\]

with \( da/dN \) in m/cycle and \( \Delta K \) in MPa\( \sqrt{\text{m}} \). The yield stress of the material is \( \sigma_y = 400 \text{ MPa} \) and the fracture toughness \( K_C = 70 \text{ MPa}\sqrt{\text{m}} \). The stress intensity factor for a crack emanating from a hole is given by

\[
K = Q \sigma \sqrt{aR}.
\]

Approximate values of \( Q \) are shown in the following table.

<table>
<thead>
<tr>
<th>( a/R )</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>&gt;5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>1.31</td>
<td>1.12</td>
<td>1.03</td>
<td>0.93</td>
<td>0.88</td>
<td>0.82</td>
<td>0.75</td>
</tr>
</tbody>
</table>
2. Solution

Failure of the plate will be assumed either by fracture, i.e. the maximum SIF reaches its critical value or by net-section yielding, i.e. when the net-section stress reaches the yield stress of the material. If $t$ is the thickness of the plate and $a_1$, $a_2$ are the left and right crack lengths, respectively, then the condition for net-section yielding is

$$\frac{\sigma_y}{wt} = \frac{\sigma_{\text{max}}}{(w - 4R - a_1 - a_2)t}$$

or

$$a_1 + a_2 = (1 - \sigma_{\text{max}} / \sigma_y)w - 4R = (1 - 130/400) \times 80 - 4 \times 3 = 42 \text{ mm}$$

To evaluate crack growth, an incremental approach will be employed. The cracks are assumed to grow at a constant rate for every $\Delta N = \text{10000 cycles}$. For every increment, the fatigue crack growth rates at the two tips will be evaluated and the cracks will be assumed to grow by

$$\Delta a_1 = \left( \frac{\text{da}}{\text{dN}} \right)_1 \times \Delta N \quad \text{and} \quad \Delta a_2 = \left( \frac{\text{da}}{\text{dN}} \right)_2 \times \Delta N.$$