Problem 55: Critical Crack Opening Displacement *

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1. Problem

A three-point bend specimen with $S = 25 \text{ cm}$, $W = 6 \text{ cm}$, $a = 3 \text{ cm}$, and $B = 3 \text{ cm}$ is used to determine the critical crack opening displacement $\delta_c$ of a steel plate according to the British Standard BS 5762. The load versus crack mouth displacement ($P-V$) record of the test is shown in Figure 1. Determine $\delta_c$ when $E = 210 \text{ GPa}$, $\nu = 0.3$ and $\sigma_y = 800 \text{ MPa}$ for steel.

![Figure 1. Load-crack mouth displacement (P-V) record of a three-point bend specimen.](image)
Determination of the critical crack opening displacement is the subject of the British Standard BS 5762 [1]. We use the edge-notched three-point bend specimen which was described in Problem 34 to determine the fracture toughness $K_{ic}$. The specimen thickness $B$ is taken about equal to the application thickness, and the beam width $W$ is twice the thickness ($W = 2B$). The specimen is fatigue precracked as in the $K_{ic}$ standard test, with the exception that a straight starter notch is recommended rather than a chevron notch.

The load versus crack mouth displacement is recorded from the experiment. Clip gages are usually installed at a distance $z$ from the specimen surface. The load-displacement records fall into the five cases shown in Figure 2. Four categories of crack-tip opening displacement are defined in relation to Figure 2: $\delta_e$ at the onset of unstable crack growth (case I) or pop-in (case II), when no stable crack growth is observed; $\delta_u$ at the onset of unstable crack growth (case III) or pop-in (case IV) when stable crack growth takes place before instability; $\delta_i$ at the commencement of stable crack growth (cases III, IV and V); $\delta_m$ at the maximum load $P_m$ (case V) when it is preceded by stable crack growth.

The critical crack-tip opening displacement $\delta$ ($\delta_n$, $\delta_o$, $\delta_u$ or $\delta_m$) is determined from the test record by

$$\delta = \delta_e + \delta_p \quad (1)$$

where

$$\delta_e = \frac{K_i^2(1-v^2)}{2\sigma_y E}, \quad \delta_p = \frac{V_p r b}{r b + a + z} \quad (2)$$

Here $V_p$ is the plastic component of the measured displacement $V$ (Figure 2) and the quantities $r$, $b$, $a$ and $z$ are shown in Figure 3.

In Equation (1) the crack opening displacement $\delta$ is equal to elastic $\delta_e$ plus the plastic $\delta_p$ contribution. The elastic part $\delta_e$ is calculated according to from the Dugdale model [2] which is modified for plane strain and by a plastic constraint factor equal to 2. The plastic part $\delta_p$ is obtained by assuming that the crack ligament $b = W - a$ acts as a plastic hinge, with a rotation point at a distance $rb$ from the crack tip. Experiments show that the value of the rotation factor $r$ lies between 0.33 and 0.48. A nominal value of 0.4 is used for the standard test. Thus, $\delta_p$ from Equation (2) with $z = 0$ becomes