ON A RELATIONSHIP BETWEEN COD AND STRAIN ENERGY DENSITY IN A MODERN MMC

N. P. ANDRIANOPoulos

Dept. of Mechanics,
National Technical University of Athens
GR-157 73 Athens, Greece

Abstract. A relationship between the critical value of Crack Opening Displacement and the mean value of dilatational strain energy density, $T_v$, computed along crack-lips, is indicated in the present work. Namely, $T_v$ takes a maximum value at the moment of crack initiation. This relationship does not hold when the crack tends to be parallel to loading axis and shear stresses dominate over normal ones. Experimental evidence from a metal matrix composite supports the present conclusions.

1. INTRODUCTION

Probably, the most attractive for practical applications crack initiation criterion is that of Crack Opening Displacement (COD), stating that a crack initiates when the opening of the crack-lips measured near its tip takes a critical value. However, and in spite of four decades been passed since Wells [1] introduced COD, doubts are still expressed regarding both the physical meaning and the accurate definition of COD [2]. In addition, limited experimental information is available concerning the behavior of COD versus crack inclination [3]. This information indicates that COD is quite sensitive to crack-inclination angle, $\beta$, deteriorating its service as a critical failure parameter.

Thus, it seems interesting an attempt to connect COD with other purely geometrical characteristics of the crack like its surface or volume and compare their behavior with mechanical quantities playing role in crack initiation.

2. SOME THEORETICAL CONSIDERATIONS

Consider a linear elastic plate containing a straight crack of length $2a$, inclined by an angle, $\beta$, with respect to the uniaxial loading axis, Fig.1. When the external stress, $\sigma_{\infty}$, is applied the crack becomes a thin elliptical hole with semiaxes, $a$ and $b$ (Fig.2). A convenient technique to obtain COD is to measure it at a certain distance, $c$, from the crack tip. Usually, $c$ is a portion (0.1–0.5) of the half crack length, $a$. Assume,
then, that for various crack inclinations, $\beta$, a set of critical COD-values, $\delta$, and fracture stress, $\sigma_{\infty}$, is obtained.

For an elliptical crack with major semiaxis, $a$, and passing through the point $(x, y) = (ca, \delta)$, elementary calculations indicate that the minor semiaxis equals to:

$$b = \frac{\delta}{\sqrt{c(2-c)}}$$

and the surface, $S$, of the crack in the xy-plane is:

$$S = \pi ab = \frac{\pi a \delta}{\sqrt{c(2-c)}}$$

In order to evaluate the crack volume, $V$, it is necessary to estimate the plate-thickness along crack-lips. It is expressed as:

$$\bar{w} = w_0 (1 + \varepsilon_{zz})$$

where $w_0$ is the initial plate thickness, $\bar{w}$ is the mean value of thickness along crack-lips and $\varepsilon_{zz}$ is the mean value of the strain normal to the plate thickness. Plate thickness being known, the volume of the crack is:

$$V = S \bar{w} = Sw_0 (1 + \varepsilon_{zz})$$