USE AND STRUCTURE OF SLEPIAN MODEL PROCESSES FOR PREDICTION AND DETECTION IN CROSSING AND EXTREME VALUE THEORY*

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SUMMARY. A Slepian model is a random function representation of the conditional behaviour of a Gaussian process after events defined by its level or curve crossings. It contains one regression term with random (non-Gaussian) parameters, describing initial values of derivatives etc. at the crossing, and one (Gaussian) residual process. Its explicit structure makes it well suited for probabilistic manipulations, finite approximations, and asymptotic expansions.

Part of the paper deals with the model structure for univariate processes and with generalizations to vector processes conditioned on crossings of smooth boundaries, and to multiparameter fields, conditioned on local extremes or level curve conditions.

The usefulness of the Slepian model is illustrated by examples dealing with optimal level crossing prediction in discrete and continuous time, non-linear jump phenomena in vehicle dynamics, click noise in FM radio, and wave-characteristic distributions in random waves with application to fatigue.

1. INTRODUCTION

The object of study in this paper is the Slepian model process, so named after D. Slepian, who introduced it (1963) to represent a normal process after zerocrossings. It describes in functional form the stochastic behaviour of a normal (stationary or non-stationary) process near a general level or curve crossing. It contains one regression term with random non-normal parameters, describing in-
itial values, such as derivatives etc., at the crossing, and one
normal residual process. Its explicit structure makes it well
suited for probabilistic manipulations, finite approximations, and
asymptotic expansions.

The paper will present the general model structure for uni-
variate processes, and generalizations to vector processes con-
ditioned on crossings of smooth boundaries, and to multiparameter
fields, conditioned on local extremes or level curve conditions.
The Slepian model is also used to derive optimal level crossing
predictors for continuous time and discrete time processes, and to
study wave-characteristic distributions in random waves with appli-
cation to fatigue.

To further motivate the study, and point at the types of con-
clusions which can be drawn, we shall briefly mention two non-triv-
ial examples, the first of which has its roots in a classical ap-
plication of crossing theory in telecommunication.

EXEMPLARY 1.1 (Clicks in FM radio) In FM radio a signal \( S(t) \) is
transmitted as an argument modulated cosine wave \( A \cos(\omega_0 t + S(t)) \),
which is corrupted by additive noise \( \xi(t) \) during transmission.
In the receiver which ideally should reconstruct \( S(t) \), the noise
causes more or less serious disturbances in the output signal. One
type of such disturbances is the so called FM click, noticed as a
short spike with high amplitude in the audible signal.

The clicks can be described statistically by means of cross-
ings in a certain bivariate noise process \( (\xi_1(t), \xi_2(t)) \), which is
a function of the original noise process \( \xi(t) \); see [13]. A click
occurs any time \( \xi_1(t) \) crosses the zero level, under the extra
condition that \( \xi_2(t) \) takes a value greater than the carrier am-
plitude \( A \). By modelling both \( \xi_1(t) \) and \( \xi_2(t) \) near such con-
ditioned crossing points by Slepian model processes one obtains
simple functional expressions for the random click shape and ampli-
tude.

In particular, if signal is absent, i.e. \( S(t) = 0 \), the clicks
are approximately of the form \( A\Psi(At) \) for large \( A \), where \( \Psi \) is
a standardized click form given by the random function

\[
\Psi(t) = \frac{\zeta_2(\xi + t^2/2)}{(-\xi + t^2/2 + \xi_1t)^2 + \zeta_2^2t^2}
\]

Here \( \xi, \xi_1, \xi_2 \) are independent random variables with exponential,
normal, and Rayleigh distributions, respectively, taking different
(independent) values at each new click. Figure 1, taken from [8],
shows examples of typical click forms for a few outcomes of the