We consider the class of stationary stochastic processes whose margins are jointly min-stable. We show how the scalar elements can be generated by a single realization of a standard homogeneous Poisson process on the upper half-strip $[0,1] \times \mathbb{R}$ and a group of $L_1$-isometries. We include a Dobrushin-like result for the realizations in continuous time.

1. Introduction.

Multivariate extreme value distributions and their domains of attraction have been studied extensively. See de Haan & Resnick (1977). For a general source on extreme value theory and applications see the book by Galambos (1978). We are not concerned here with domains of attraction. The univariate limiting extreme value types can be transformed into one another by means of simple functional transformations ($\log x, 1/x, x^a$ etc.). The same is true for multivariate extreme value distributions. That is a distribution is determined by its margins and independently, by its dependence function. The choice of marginal type is one of convenience. We use the negative exponential family. That is we use $X$ which is such that $-\log p(X > x) \equiv x/EX$. Notice that these are limiting distributions of smallest values.

We say that a random vector $Z$ with elements $Z_k$, is "min-
stable" if and only if it is a limiting extreme value distribution with negative exponential margins. In fact \( Z \) is min-stable if and only if \( \min (Z_n/a) \) has a negative exponential distribution for any nonrandom vector \( a \), with elements \( a \in [0, \infty) \) at least one of which is positive. Notice that if \( a_n = 0 \), then \( Z_n/a = \infty \) and the term plays no role in the minimization.

In Section 2, we present a representation for any finite or infinite dimensional min-stable random vector \( Z \). It depends upon a standard homogeneous Poisson process on the strip \([0,1] \times \mathbb{R}_+\) and a set of non-random functions \( f_n: [0,1] \rightarrow \mathbb{R}_+ \) which correspond to the components \( Z_n \) of \( Z \). We consider the nonuniqueness of \( \{f_n\} \) and introduce the group of "pistons": a class of function transformations \( \Gamma \) which are such that \( \int_0^1 f(f)(t)dt = \int f(t)dt \) for all non-negative \( f \). We discuss this group in Section 3. In Section 4 we consider the implications of strict stationarity. We consider continuous time stationary processes in Section 5. Finally we have a general discussion in Section 6. The proofs and detailed derivations can be found in De Haan (1983) and De Haan and Pickands (1983).

2. Representation.

Let \( \{S_n, U_n\} \in [0,1] \times \mathbb{R}_+ \) be the points of a standard homogeneous Poisson process. Let the random vector \( Z \), with elements \( Z_n \), be min-stable in our sense. Then there exists a non-unique sequence of functions \( f_n: [0,1] \rightarrow \mathbb{R}_+ \) such that \( \{Z\} \) and \( \bigoplus_{l=1}^N (U_l/f(S_l)) \) have the same joint distribution. Since the distributions are the same we can without loss of generality let

\[
Z_n \equiv \bigoplus_{l=1}^N (U_l/f_n(S_l))
\]

for all \( n \). For \( z_n \in (0,\infty) \), the event

\[
\{Z_n > z_n\} = \bigoplus_{l=1}^N \{U_l/f_n(S_l) > z_n\} = \bigoplus_{l=1}^N \{U_l > z_n f_n(S_l)\}
\]

\[
\equiv \text{Emp}\{s, u \mid u \in [0, z_n f_n(s)]\}
\]

where \( \text{Emp} A \) means that \( A \subset [0,1] \times \mathbb{R}_+ \) contains no points of the homogeneous Poisson process on \([0,1] \times \mathbb{R}_+\). But the number of points in the process in \( A \) has the Poisson distribution with mean (parameter) \( \lambda_2(A) \) where \( \lambda_2 \) is 2-dimensional Lebesgue measure. It follows that

\[
-\log p(Z_n > z_n) \equiv \lambda_2 \{s, u \mid u \in [0, z_n f_n(s)]\}
\]