Microcrack initiation at tip of a rigid line inhomogeneity

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Received 20 May 1996; accepted in revised form 25 October 1996

Abstract. A dislocation emission mechanism for microcrack initiation at tip of a rigid line inhomogeneity is proposed in the present paper. For a rigid line inhomogeneity embedded in a ductile matrix, it has been observed that dislocations of one sign are driven away from the tip due to high stress level; while the stationary dislocations of the opposite sign are left behind near the tip of the line inhomogeneity. As the result, a Zener–Stroh crack is initiated at the tip of the inhomogeneity. A very interesting and important result that emerged from the analysis is that the critical stress intensity factor for a line inhomogeneity can be related to the fracture toughness of a crack in the same material.

1. Introduction

A rigid line inhomogeneity embedded in an elastic material is of theoretical interest because it is the counterpart of conventional crack in solids. That is why it is called ‘anticrack’ by some researchers [1]. Besides the theoretical importance, it has been taken as an analytical model for certain composite materials, such as metal matrix composites where the reinforcement phases are made of ceramics and the matrices are metals. The big difference in moduli between ceramics and metals makes the rigid line inhomogeneity model a very good approximation. According to a near tip asymptotic expansion, we know that the stress field has a square-root singularity at the inhomogeneity tip. It is not strange that fracture mechanics analyses for a line inhomogeneity have been carried out by solving whole field solutions for various configurations. Readers are referred to Hasebe et al. [2–3] for earlier works on this topic. Recent progress on a line inhomogeneity embedded in anisotropic elastic matrix was given by Li and Ting [4] and Fan and Keer [5]. All the works in the literature led to fracture criteria, such as stress intensity factors which are in analogue to linear fracture mechanics for a crack, and crack initiation direction from the tip of the line inhomogeneity.

It is noted that all the above mentioned investigations for the anticrack are merely a kind of solving the boundary value problem. Based upon those analyses, a criterion of crack initiation at the inhomogeneity tip, analogue to a conventional crack propagation, was formally set as

\[ K_1^* = K_{IC}^* \]  

(1.1)

where the * is used here specially for anticrack in order to distinguish the conventional crack stress intensity factor. The critical value at the right-hand side of (1.1) is deemed to be a material constant which should be determined from a test. However, from the authors’ knowledge, there is no such experimental result in open literature. It is our conjecture that this constant can be related to fracture toughness \( K_{IC} \) for a crack in the same material since both cases associate with the square root singularity. Nonetheless, correlating these two critical values is not a straightforward matter. Apparently, the Griffith energy release rate concept
cannot be applied here since the crack initiation ahead of the line inhomogeneity is not a self-similar process.

In order to search for a possible connection between $K_{IC}$ and $K^*_c$, for instance

$$K_{IC} = C K_{IC}, \quad (1.2)$$

where $C$ is to be determined constant, a model of crack initiation ahead of the line inhomogeneity is proposed in the following sections. The mechanism of the model is schematically shown in Figure 1(a), (b) and (c). Let us consider a semi-infinite long line inhomogeneity loaded by a $K^*_c$-field as shown in Figure 1(a). Dislocations are driven away from the tip due to high shear stress along certain slip planes (Figure 1(b)). Since the dislocations are generated in pairs with opposite signs, dislocations with the sign opposite to those driven away pile up at the tip of the inhomogeneity. The present situation is different from that of crack-tip dislocation emission where the left dislocations entered the main crack and led the crack tip blunting [6]. For the current case, strain energy is built up as the number of dislocations piled up against the line inhomogeneity tip increases. One of the possible ways of releasing the high strain energy caused by this dislocation pileup is to initiate a microcrack ahead of the line inhomogeneity (Figure 1(c)). It is important to point out that this crack-initiation mechanism was exactly observed by Kikuchi et al. [7] and was called anti-Zener-Stroh crack’ by Weertman [8] later on.

We are fully aware of other possible mechanisms to initiation cracks in front of the inhomogeneity: e.g., a void initiation and evolution model ahead of a crack tip proposed recently by Shih and his co-workers [9]. Other failure modes, such as interfacial cracking at matrix/inhomogeneity interface, are also possible if they are preferred in comparison with other mechanisms. However, it is our intention to analyze the above-mentioned model which has been experimentally confirmed [7], rather than to compare all the possible failure mechanisms. After all, this is the first analytical work for the crack initiation for this configuration according to the authors' knowledge.

2. Assumptions and simplifications

In the following formulation, we made some assumptions in order to simplify the problem without losing key components in the model. First, we assume that the as initiated microcrack in Figure 1(c) is still under the $K^*_c$-field control. This allows us to discuss a generic solution in terms of the stress intensity factor rather than a limited geometry and loading configuration. However, it is not difficult or complicated to solve a finite crack initiated ahead of the inhomogeneity tip where the whole field solution of the problem is applied rather than a $K^*_c$-field solution. The validity of this assumption is confirmed by the later formulation and numerical results discussed in the last section.

Secondly, let us examine forces acting on the dislocations driven away from the inhomogeneity tip to ensure that the proposed mechanism is feasible. Referring to Figure 1(c), there are three forces acting on the dislocation at point A:

$$f_\sigma = \frac{K^*_c b_0}{\sqrt{2\pi L}} g(\theta), \quad (2.1)$$

the force due to the singular stress field;

$$f_r = C_r \frac{\mu b_0^2}{L}, \quad (C_r \text{ is a non-dimensional constant}), \quad (2.2)$$