Post-yield fracture mechanics analysis of the combined thermal and mechanical loading of a centre-cracked plate

T.K. HELLEN * and W.S. BLACKBURN **

* Central Electricity Generating Board, Berkeley Nuclear Laboratories, Berkeley, Glos, GL13 9PB, U.K.
** Central Electricity Generating Board, S.E. Region SSD, Gravesend, Kent, DA12 2RS, U.K.

(Received 24 July 1985; in revised form 20 August 1986)

Abstract

The elastic-plastic stress analysis of a centre-cracked plate subjected to two types of loading is considered using the non-linear BERSAFE finite element system. The loading includes both uniaxial tension and a quadratic thermal gradient across the plate, both of which independently would tend to open the crack. In the present work the combined effect of these two load forms is investigated by applying one after the other using the restart facility. In the case of the thermal load followed by the tensile load, several restarts of the tensile part were made at successively lower thermal loads. Also, both combinations of load application were applied to a plate initially uncracked, then the crack was opened from the loaded state by using a nodal load release technique. In all cases, path independent contour integrals were calculated to assess the fracture conditions at the crack tip under the above styles of loading, particularly with a view to supplying results which can be compared to proposed CEGB fracture procedures.

1. Introduction

Post-yield fracture mechanics procedures are now well known for test cases subjected to straightforward loading systems such as tension and three point bending. Semi-analytical techniques are available to calculate fracture parameters, such as $J$-integrals or crack opening displacements, as a function of stationary or propagating cracks. For more complicated geometries and more complicated loadings, such techniques become more and more difficult to apply and so recourse to finite element methods, which can deal with any arbitrary type of geometry and loading, is necessary.

A new release of the non-linear version of BERSAFE [1] enables such analyses to be conducted, and using the post-processor program PLOPPER [2] $J$-type integrals, designated $J^*$ and $J^R$, may be evaluated. In addition, the potential energy release rate may be evaluated directly using the virtual crack extension concept and two full or substructured runs. The new plasticity solutions include initial stress and tangent stiffness techniques, the latter enabling higher rates of convergence as plasticity dominates albeit at the cost of extra computation and the risk of divergence [3]. The latter effect can be automatically removed, upon detection, using one of several strategies, including partial tangent stiffness whereby the tangent slope is relaxed to lessen the divergence probability, by a factor $\mu$, $0 < \mu > 1$: $\mu = 0$ gives initial stress, $\mu = 1$ gives full tangent stiffness.

The present analyses utilise these calculation aids to deal with the centre-cracked plate subjected to both tensile mechanical and thermal loading, each restarted after the other, including the effects of loading the uncracked plate then releasing nodal forces to simulate the sudden appearance of a crack. The combined, final, results are investigated to see if the loading path is important, and also to assess the viability of current CEGB fracture procedures known as R6 [4] in such cases.
2. Parameters for post-yield fracture mechanics

For post-yield fracture mechanics, the parameter most useful for fracture assessment is the potential energy release rate at the crack tip under consideration. The two main techniques available in finite elements are the direct potential energy difference evaluation and contour integration [5]. For the former, the potential energy, \( P \), is accumulated at any load level in the incremental elastic-plastic process as the strain energy less the potential of forces, with a given crack length \( a \). Then, the calculation is repeated with a small crack extension \( \delta a \), such that \( \delta a \) is small compared to the size of the elements around the crack tip, and with identical load increments, to give potential energy \( P + \delta P \). Then, at any load level, the potential energy release rate \( G \) is given by \( G = \delta P / \delta a \). Accurate results are obtained even with values of \( \delta a \) several orders of magnitude smaller than \( a \).

This technique requires, nominally, two computer submissions differing only by the crack length increase, but has the advantage of being readily applicable to new types of problems (e.g. three-dimensions, shells, new elements) and any type of applied load. Use of substructuring for near tip elements is possible to avoid complete recomputation. Alternatively, the more sophisticated virtual crack extension techniques [6] are possible which further minimise the number of elements involving a second calculation.

Contour integration is based on the \( J \)-integral of Rice [7], which is

\[
J = \int_C W \, dx_2 - T_i \frac{\partial u_i}{\partial x_1} \, ds
\]

(1)

integrated around a contour \( C \) surrounding the crack tip, where at any point on the contour \( T_i \) and \( u_i \) are components of traction and displacement, and \((x_1, x_2)\) are local axes such that the origin is at the tip and the crack extends along negative \( x_1 \). \( W \) is the strain energy density defined by

\[
W = \int \sigma_{ij} \, d\epsilon_{ij}
\]

(2)

where \( \epsilon_{ij} \) is the sum of elastic and plastic strains at any point. As long as this integration is conservative, the stress is a function of strain alone, \( J \) is path independent provided the crack tip is the only singularity within the contour. This applies to both elastic and non-linear elastic conditions. In general incremental non-conservative plasticity the integral is path dependent. Since the theoretically correct contour is of zero length at the crack tip, one would expect the results from contours near the tip to be more accurate than those further away, but in practice they are not because the tip region displacements and stresses are relatively inaccurate. It is usually found, however, that \( J \) remains sufficiently constant over a range of contours throughout the mesh under the conditions when the integral is valid.

These conditions require two-dimensional linear or non-linear behaviour with primary loading. For more complex structures and loadings, such as thermal, alternative forms of integral are required. A family of such integrals has been implemented in the post-processor program, PLOPPER, to the BERSAFE system [2]. Of particular interest to the application to mechanical and tensile loading are the integrals

\[
J^* = \lim_{\rho \to 0} \int_{\rho} \frac{1}{2} \sigma_{ij} \frac{\partial u_i}{\partial x_j} \, dx_2 - T_i \frac{\partial u_i}{\partial x_1} \, ds
\]

(3)

and

\[
J_W^* = \lim_{\rho \to 0} \int_C W \, dx_2 - T_i \frac{\partial u_i}{\partial x_1} \, ds
\]

(4)