Plane stress dynamic plastic field near a propagating crack tip

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Abstract. In this paper the asymptotic solution to the plane stress dynamic plastic field surrounding a propagating crack tip is given. The perfectly plastic model and Mises yield condition as well as J2 flow theory are adopted. The force of inertia is considered in the equations of motion. The asymptotic expansions of displacements, strains and stresses are given and solved for the dominant terms. The results show that strain possesses ln (A/r) singularity at the crack tip. Finally, the stress distribution surrounding the crack tip are given numerically.

1. Introduction

The crack-tip fields for a plane stress case are quite different from that for a plane strain case. For a plane strain problem, the asymptotic solutions to the quasi-static growing crack are given by Slepyan [1], Amazigo and Hutchinson [2], Gao [3, 4], Rice [5] and Gao and Hwang [6]. As for the plane stress problem, the quasi-static growing crack is very mysterious. Many authors have tried to give the asymptotic solution but none have been successful. Gao and Hwang [7] give a systematic analysis considering the strict continuity conditions between different domains, but finally, there is no reasonable solution to be found. When the quasi-static assumption is abandoned, the dynamic solutions to the crack-tip field that are obtained [8, 9, 10] appear to be quite different from the quasi-static ones, and the quasi-static solutions cannot be obtained from the limit case of $M = 0$; here $M$ is the Mach number of the crack. In [11] the contradictions in the quasi-static solution of the mode I crack are revealed. The analyses of [11] show that when the crack tip is approached the inertia force possesses higher order of singularity so that it cannot be neglected from the equations of motion, no matter how small the Mach number is. Thus, for a plane strain problem, the important influence of inertia force on the solutions of crack-tip fields has been demonstrated. For the plane stress problem, we do not even have the quasistatic solution to the mode I crack-tip field. Therefore the dynamic solution will be more interesting for the plane stress case.

2. Basic equations

2.1. Statement of the problem

Let $x_i (i = 1, 2, 3)$ be the fixed rectangular Cartesian coordinate system and $t$ denotes time. We consider a crack propagating in $x_1$ direction at a constant speed $V$. Let $x, y, z$ be the coordinate system moving with the crack tip, (see Fig. 1) so that
then the steadily propagating crack means that for any quantity $\Phi$ we have,

$$
\frac{\partial \Phi}{\partial t} + V \frac{\partial \Phi}{\partial x} = 0
$$

or

$$
\frac{d\Phi}{dt} = -V \frac{\partial \Phi}{\partial x}
$$

and for any tensor $T^{ij\ldots k}$ we have

$$
T^{ij\ldots k} e_i e_j \ldots e_k = -V \frac{\partial}{\partial x} (T^{ij\ldots k} e_i e_j \ldots e_k).
$$

2.2. Equations of motion

Let $r, \theta$ be the polar coordinate system which has the common origin with $x, y$ system at any fixed moment $t$, then the equations of motion can be written as

$$
\begin{align*}
\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{r\theta}}{r \partial \theta} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) &= \rho w_r, \\
\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{\partial \sigma_{\theta\theta}}{r \partial \theta} + \frac{2}{r} \sigma_{r\theta} &= \rho w_{\theta}.
\end{align*}
$$

in which $\sigma_{\alpha\beta} (\alpha, \beta = r, \theta), w_z, \rho$ denote stress, acceleration, and mass density respectively.