Effect of microcrack array on stress intensity factor of main crack

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Abstract. An improved technique for solving crack-microcrack interaction problems is developed, based on the Green’s function for a dislocation dipole placed in the vicinity of a main crack. The microcrack array is characterized by the microcrack density, orientation and length distributions. The stress field near the main crack tip is taken as a sum of a singular term of an anisotropic problem with stress intensity factor \( K \), and a regular term. \( K \) and the regular term are determined from a system of equations obtained by self-consistency arguments. The proposed technique is illustrated by numerical solutions of several interaction problems for particular configurations of microcrack array.

1. Introduction

Three approaches have recently been proposed for evaluating elastic fields resulting from an array of microcracks in the vicinity of a main crack. One idea is to model microcrack array by inclusion of an effective elastic media. The effect of crack tip shielding produced by this kind of 'softer inclusion' has been well addressed in various publications [1–4]. This approach has several shortcomings. First, it does not take into account local fluctuations of microcrack arrangements, which can be critical for crack stability problems. If the microcracks are then not homogeneously distributed in the array, an equivalent elastic inclusion should be non-homogeneous and anisotropic, which leads to computational difficulties. Moreover, unknown relationships between the statistics of the microcrack array and the effective elastic constants in the crack-microcrack interaction constitute certain limitations to the effective elastic inclusion method. A second approach to crack-microcrack interaction is based on a detailed description of the location, size and orientation of each microcrack in every particular realization of a random microcrack array [5–10]. Apparently, this leads to a computational limitation similar to that in conventional molecular dynamics. A third approach implements a statistical mechanics technique, describing a microcrack array in terms of statistical distributions of microcrack densities, sizes and orientations. This leads to evaluating integral parameters associated with microcrack array [7, 11–13, 18].

The present work follows the last approach and continues our previous work [11–13]. The solution to the problem of crack-microcrack interaction is based on the Green’s function of a dislocation dipole placed in the vicinity of the main crack tip [14]. Elastic fields which result from various orientations and sizes of microcrack array are discussed for a special case of prescribed microcrack opening. The interaction between a microcrack array having all microcracks parallel to the main crack and the main crack is considered, in order to illustrate shielding or amplification effects. The stress field near the main crack tip is taken as a summation of two terms. The first term represents a singular field near the crack tip for an anisotropic elastic material with stress intensity factor (SIF) \( K \). The second term represents a regular field, and is determined from a system of equations expressing a self-consistency
condition. The shielding effect of a microcrack array is discussed for uniform and nonuniform microcrack distributions. The solution for a uniform microcrack distribution is compared with that obtained by an effective elastic inclusion model [3].

2. Crack-microcrack array interaction

2.1. Formulation of the problem

The linear elastic interaction of a crack and microcrack array can be obtained by a superposition method using the Green’s function $G$ for a dislocation dipole interacting with a crack. Microcrack opening displacement is conventionally represented by a continuous distribution $b(\xi)$ of dislocation dipoles. Thus the displacement $u(x)$, the stress $\sigma(x)$ and the stress intensity factor $K$ at the main crack caused by a particular microcrack ($l_0$) can be expressed as

$$u(x) = \int_{l_0} b(\xi) \Phi(x, \xi) d\xi,$$

$$\sigma(x) = \int_{l_0} b(\xi) F(x, \xi) d\xi,$$

$$K = \int_{l_0} b(\xi) G_{SIF}(\xi) d\xi,$$

where the influence functions $\Phi$, $F$ and $G_{SIF}$ are expressed in terms of the Green’s function (Appendix 1). By means of superposition, the stress $\sigma^A$, displacement $u^A$, and stress intensity factor $K^A$, caused by the microcrack array can now be obtained by integrating $b$ over $V$ with the microcrack density $\rho$ as a weight function, similar to (1) [11–13]. The resulting equations are

$$u^A(x) = \int_V \rho(\xi) b(\xi) \Phi(x, \xi) d\xi,$$  \hspace{1cm} (2a)

$$\sigma^A(x) = \int_V \rho(\xi) b(\xi) F(x, \xi) d\xi,$$  \hspace{1cm} (2b)

$$K^A(x) = \int_V \rho(\xi) b(\xi) G_{SIF}(\xi) d\xi,$$  \hspace{1cm} (2c)

where $V$ is the area of microcrack array, and the microcrack density $\rho$ has dimension $[\rho] = [\text{mm}^2]/[\text{mm}^3]$. It should be noted that $\rho$ is different from dimensionless ‘microcrack density’ $\varepsilon$ which is conventionally used in damage mechanics. Here, a corresponding quantity is microcrack concentration $\rho l$ which varies between 0.0 and 0.2 for ‘dilute’ concentrations in the two dimensional case.

As long as

$$\lim_{r \to 0} r^{-1/2} \rho(r) l(r) = 0$$  \hspace{1cm} (3)