Stress intensity factor analysis for part-elliptical cracks in structures

V.A. VAINSHTOK and I.V. VARFOLOMEYEV
Institute for Problems of Strength, Academy of Sciences of the Ukr. SSR, Kiev, USSR

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Abstract. A method based on the generalized weight function theory is used for solving three-dimensional linear elastic fracture mechanics problems. A complete system of equations of the weight function method (WFM) has been obtained for the calculation of stress intensity factors (SIF) for part-elliptical cracks subjected to arbitrary normal loading.

A procedure of the WFM is described to analyze structural components containing surface (semi-elliptical), corner (quarter-elliptical) and embedded (elliptical) flaws. The efficiency of the proposed method is illustrated by solving a number of methodical problems.

1. Introduction

The importance of the development of the effective methods for the stress intensity factor (SIF) analysis stems from the necessity for fracture mechanics applications to solving practical problems. The possibilities offered by the finite element method (FEM), boundary integral equation method (BIEM) and others do not always satisfy the user's requirements because of an appreciably high cost and tediousness of computations even for a single configuration of a cracked body. The use of conventional numerical methods becomes inefficient in the case of repeated analyses of cracks in structures (for instance, in fatigue crack growth prediction, in design, etc.). In this case, engineering methods of the SIF calculation are used rather than FEM or BIEM.

The engineering methods of the SIF calculation can be efficient when the following conditions are satisfied:

1. The SIF calculation is based on the results of the stress analysis in an uncracked body;
2. The time for preparing the problem and computer time are comparatively short;
3. The results can be obtained for several crack types;
4. Essential results obtained in expensive unique computational experiments (most commonly for the uniformly loaded cracks [1–4]) are widely used;
5. For a crack subjected to an arbitrary nonuniform loading, the error of the SIF estimates ranges from 5 to 10 percent.

The weight function method (WFM) satisfies the above requirements.

The methods of computing the weight functions for two-dimensional crack problems are based on the works by Bueckner [5] and Rice [6]. The analysis of three-dimensional cracks using the WFM generally involves the averaged weight functions [7–9]. This approach allows the averaged SIF values to be determined at the crack front points coinciding with the ellipse vertices.
To calculate the SIF distribution along the crack front, an approach was proposed in [10–13] based on the use of the generalized weight functions which are proportional to the derivatives of the crack opening displacement field with respect to the arbitrary configuration parameters varying with the crack front translation. The application of the generalized weight functions is particularly useful for solving complex multi-parametric fracture mechanics problems, for analyzing the SIF calculation error, as well as for the estimation of the SIF variation due to the crack shape change [12].

The paper presents the procedure of the generalized weight functions application for the mode I SIF computation along the front of arbitrary loaded part-elliptical cracks.

2. Flaws schematization in structures

Single and group flaws in structural components can be substituted by part-elliptical cracks in accordance with the principles presented, for instance, in [14, 15]. In this case, surface, corner and internal flaws are reduced to semi-elliptical, quarter-elliptical and elliptical cracks, respectively. In addition, in the SIF computations real structural components are substituted by bodies of more simple configuration, say by plates, while actual load is reduced to stresses applied to the crack faces.

Let the geometry of the body be determined by a set of independent parameters $\xi_i(\xi_i, i = 1, m)$ which can vary with the crack front translation. In particular, for a plate with a surface or a corner crack (Figs. 1a, b), we assume

$\xi_1 = \alpha = a/b, \quad \xi_3 = \beta = a/h, \quad \xi_3 = \gamma = 0, \quad \xi_4 = h/W, \quad \xi_5 = \gamma = 0, \ldots$

for an embedded crack (Fig. 1c)

$\xi_1 = \alpha = a/b, \quad \xi_2 = \beta = a/d, \quad \xi_3 = \gamma = e/h, \quad \xi_4 = h/W, \quad \xi_5 = \gamma, \ldots$

Here $a$ and $b$ are the ellipse half-axes; $h$ is the body thickness for a surface and a corner crack; $2h$ is the body thickness for an embedded crack; $d$ and $e$ are the distances from the ellipse centre to the body front surface and the axes of symmetry, respectively; $W$ is the width (half-width) of the plate with a corner (surface, embedded) crack; $\gamma$ is the angle between the ellipse longer axis and the body boundary. The parameters which define the distances between cracks, the crack shape deviation from part-elliptical, the body surface curvature, etc. can also be used as $\xi_i$.

3. A system of equations of the weight function method for the stress intensity factor analysis

Consider a linear elastic body containing a part-elliptical crack of area $S$ and contour $\Gamma$. The polar $(r, \theta)$ and parametric $(\rho, t)$ coordinate systems with the origin in the centre of the ellipse are used along with the Cartesian coordinates $(x, y)$. The relationship between the polar and parametric coordinates is established as

$$\tan \theta = \alpha \tan t, \quad \rho = r/R, \quad R = a(\sin^2 \theta + x^2 \cos^2 \theta)^{-1/2},$$ (1)