Convenient closed form stress intensity factors for common crack configurations

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ABSTRACT

Linear elastic crack-tip solutions for twelve different shapes of cracked body of interest, are given. The purpose is to provide efficient "closed" formulations of data previously presented in a tabular or graphical manner. The formulae assist the user of fracture mechanics in that they carry out the interpolative step accurately and therefore may be usefully incorporated in other crack computational procedures, such as fatigue crack growth prediction, crack-tip plasticity corrections, etc. The method used to generate the formulae can be applied to other cracked body geometries.

1. Introduction

The feature of fracture mechanics which is of most use to the practising engineer is that it provides a way of characterising (describing or identifying) near-crack-tip states. If appropriate solutions are available, differently cracked bodies can be compared although the overall geometries, the crack lengths and the loadings may be unlike. In particular one may compare the crack propagation behaviour of a laboratory or "prototype" cracked body and the behaviour of a real structure made of the same material. In certain cases identical crack-tip states can be established in a comparison and therefore confident predictions of full-scale crack propagation behaviour can be made, although the microstructural mechanisms of the fracture process may not be fully understood.

However, as in all applications of mechanics to engineering situations, the prediction will only be good if the analysis which generated the given characterising number takes into account the real features of the problem. For cracked bodies, solutions should recognise the loading and overall geometry of the body (including the crack length), the constitutive behaviour of the material, and the triaxial nature of the crack-tip stresses. Assessment of a real structure is therefore often complicated by the difficulty in obtaining a precisely relevant solution which covers the particular combination of features present in a given practical example.

2. Using existing solutions

Even if a valid solution can be found, further difficulties often arise because many of the examples of practical interest have been analysed using numerical methods of solution which lead to the expression of results at discrete points. The resulting lack of a "closed form" requires interpolative procedures to be used and makes it awkward to incorporate these solutions in further computational routines.

For example, consider the case of a crack growing from the edge of a circular hole. The stress-field-intensity factor \( K_1 \) for such a case can be expressed as \( K_1 = \sigma \pi c F \) (see Fig. 1) and Ref. [1] provides tabular information which may be used to evaluate the number \( F \) for particular ratios of \( r/c \). If the crack size is known,
say a 2.5 mm crack is discovered at the edge of a 10 mm radius hole in a material whose toughness \((K_{IC})\) is known to be 60 MNm\(^{-3}\), it is simple to establish from [1] that \(F\) lies between 0.96 and 1.09 and that the critical fracture stress therefore lies between 329 and 302 MN m\(^{-2}\). However, more often in practice it is the stress which is known, and one wishes to predict the size of a slowly-growing or other hypothetical crack which would be critical. This is less straightforward, requiring a number of trial calculations to establish the nearest appropriate value of \(F\), and it may lead to unacceptably wide bounds on the calculated critical crack length. A specified stress of 200 MN m\(^{-2}\) for example, places the critical crack length at between 30 and 50 mm.

Lack of accuracy is more serious where fracture mechanics is used to predict fatigue crack propagation life. (Experiments have shown that the rate of propagation \((da/dn)\) can often be related to the range of stress intensity factor \((\Delta K)\), viz. \(da/dn = C(\Delta K)^m\) where \(C\) and \(m\) are empirical constants.) Although in theory one can apply an integration procedure to find the number of cycles required to propagate a crack from an initial to a critical size, in practice this requires a continuous and accurate expression for \(F\), if large cumulative errors are to be avoided.

3. Closed form presentation

The purpose of this paper therefore is to provide a catalogue of results, derived from solutions in the current literature but presented as "closed" formulae. These can be used for interpolation or inverse solution, or can be incorporated in further computations to predict fatigue crack propagation life for example, or, as is presently proposed, to estimate creep cracking life in "creep brittle" materials [2]. This article deals with linear elastic material and a later article will extend the procedure to cover elastic/plastic materials and applications.

The formulae presented here were optimised using a computer program which alters variable coefficients to minimise the sum of the squares of the errors at the data points fed in. There is no limitation on the number of coefficients or on the form of

\[\sigma\]

\[c\]

\[\tau\]

Figure 1.