THE VISCOSLY BLUNTING CRACK

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The problem of the motion of the surface of an elliptical crack in a viscous solid in plane strain was solved by Berg [1]. However Berg's solution gives information only about displacements of points in the crack surface. In order to examine the stress and strain history of points ahead of an initially sharp crack perpendicular to the applied stress, and blunting in plane strain, an iterative numerical technique has been used. This is based on the elastic solution of Inglis [2].

The small strain elastic solution may be changed to the viscous solution using the Rayleigh analogy [3], by making the material incompressible and replacing the shear modulus by the coefficient of viscosity $\mu$. The displacements $u,v$ become displacement rates $\dot{u},\dot{v}$ and the strain becomes the strain rate. The field solution was determined by a computer program on the points of a square grid in which the points were initially a distance $a_o \times 10^{-4}$ apart, $2a_o$ being the initial crack length. In a small interval of time $\Delta t$ the displacements were taken as $\dot{u}\Delta t$ and $\dot{v}\Delta t$ (neglecting higher order terms) so that the new coordinates of each point could be calculated numerically. The new semi axes of the ellipse were fed into the program and new values of the stress and displacement determined. The process was repeated several times. However, at the surface of the crack the accuracy of the resulting large displacements may be checked from Berg's finite strain solution [1]. The agreement was good, as is clear from Fig. 1 which shows the radius of curvature $\rho$ of the blunting crack tip calculated from both the incremented small strain and finite strain solutions. This agreement confirms the appropriateness of incrementing displacements in a small strain solution to obtain a large geometry change solution.

During the crack blunting process there is a large change in the crack tip radius compared with the change in $a$, and this suggested that the stress state at points $x/2\rho$ from the crack tip be examined. Although the crack blunts as an ellipse rather than a parallel-sided crack, $2\rho$ may be called a crack tip opening displacement $\delta$. In Fig. 2 the state of stress ahead of the crack has been characterized by a dimensionless parameter $\sigma_m/\sigma$ where $\sigma_m$ is the mean stress defined as

$$\sigma_m = (\sigma_x + \sigma_y + \sigma_z)/3$$

and $\sigma$ is the equivalent stress defined as

$$\sigma = \sqrt{[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2]/2}$$

the shear stresses being zero on the line of symmetry.

It will be noted that the state of stress close to the blunting crack tip can be characterized by $x/\delta$ independent of time for
\( x/\delta < 1.5 \). For comparison, Fig. 2 also shows the stress state ahead of a crack in a rigid non-hardening material in which crack blunting occurs in a log spiral slip line field, calculated from Rice and Johnson [4].

The analogy between viscous and elastic deformation has already been noted, but by putting \( n = 1 \) the viscous case may also be regarded as being a limiting case [5] of the Ludwig power law in which \( n \) is the strain hardening exponent. Thus it seems reasonable to expect that the viscous and non-hardening cases are bounds on the stress state ahead of the blunting crack, and this is confirmed by comparison with the state of stress calculated from McMeeking's [6] power hardening results.

Turning now to the strains, it should be noted that although the stresses ahead of the crack fall with time, the strains accumulate. The solution used here is of course essentially a small strain formulation, which is incremented to follow the geometry change, checking the surface displacements against an analytic large geometry change solution. Near the crack tip large strains necessarily arise even in small time increments and a small strain formulation is limited. However in order to get a feel for the strains, the strain increments have been calculated from the small strain solution thus neglecting second and higher order displacement terms. These increments were converted to increments of effective strain \( \Delta \varepsilon \), in which \( \Delta \varepsilon \) is defined from the increments of principal strain as

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\Delta \varepsilon = \sqrt{\frac{2}{9} \left[ \left( \Delta e_x - \Delta e_y \right)^2 + \left( \Delta e_y - \Delta e_z \right)^2 + \left( \Delta e_z - \Delta e_x \right)^2 \right]} \]

the shear strains being zero on the line of symmetry. The increments of effective strain were then summed. Again \( \varepsilon \) is determined by \( x/\delta \) and is independent of time close to the crack tip as is shown in Fig. 3. As the stress states for viscous and non-hardening cases have been compared it seems appropriate also to compare the strains, which are taken from the small scale yielding analyses of Rice and Johnson [3] and McMeeking [5], and these are also given in Fig. 3. The strains in the non-hardening case are greater than the viscous although the state of stress is less severe. The combination of strain and stress state ahead of blunting cracks is of particular interest as the combination of these parameters has been used in a ductile failure criterion by Mackenzie, Hancock, and Brown [7].

REFERENCES


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