Singular integral equation for antiplane-wave scattering by a semi-infinite crack

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Abstract. The problem of antiplane-wave scattering by a semi-infinite crack is reduced to a singular integral equation of the Cauchy type. This equation is obtained by treating the problem as the limiting case of a sequence of problems for which the crack-opening displacements decay exponentially at infinity, and by using real-variable (as opposed to complex-variable) Fourier-transform methods. An integral identity is used to obtain the solution of the singular integral equation. The solution is shown to coincide with the classical Wiener-Hopf solution of the problem.

1. Introduction

Several problems of elastic-wave scattering by cracks of finite lengths have been reduced to systems of singular integral equations of the Cauchy type [1-4]. For wave scattering by cracks of infinite lengths, however, no such equations have been obtained.

The usual analyses of scattering by semi-infinite cracks lead to the formulation of integral equations, but these equations are not of the singular Cauchy-type. For instance, Noble [5] (section 2.4) uses a Green-function method to analyze the problem of antiplane-wave scattering by a semi-infinite crack; he obtains an equation for the crack-opening displacement that involves both integral and differential operators, but he does not attempt to transform this equation into a singular integral equation.

In this paper, we reconsider the problem of antiplane-wave scattering by a semi-infinite crack, and we reduce this problem to a singular integral equation of the Cauchy type. This equation is obtained by treating the problem as the limiting case of a sequence of problems for which the crack-opening displacements decay exponentially at infinity, and by using real-variable (as opposed to complex-variable) Fourier-transform methods. Then we use an integral identity to obtain the solution of the singular integral equation.

In Section 2 we state the problem of antiplane-wave scattering by a semi-infinite crack and, in Section 3, we derive the corresponding singular integral equation. The solution of this equation is obtained in Section 4. We
show in Section 5 that the Wiener-Hopf method yields the same solution. Finally, the Appendix contains a proof of the integral identity that is used in Section 4.

2. Statement of the problem

Consider a linearly-elastic, homogeneous, and isotropic solid that is cracked along the positive $x_1$-axis as shown in Fig. 1. The solid is unbounded and the crack extends to infinity in the $\pm x_3$ directions.

Let $\rho$, $\mu$, and $s_T$ denote the mass density, the shear modulus, and the slowness of transverse waves in the solid. We have

$$s_T^2 = \rho / \mu.$$ 

The equations of linear elastodynamic theory for antiplane-shear (or SH) deformations are given in Achenbach [6] (p. 58). A time-harmonic SH wave is normally incident on the crack. Let $u_0$ and $\omega$ denote the amplitude and the frequency of the incident wave. The time factor, $\exp(-i\omega t)$, which is common to all field variables in a steady state regime, is omitted in the sequel. Then, the displacement generated by the incident SH wave can be written in the form

$$u_3^i(x_1, x_2) = u_0 \exp(i\omega s_T x_2). \tag{2.1}$$

In the plane of the crack ($x_2 = 0$), the stress associated with the displacement field of equation (2.1) takes the form

$$\sigma_{23}^i(x_1, 0) = i\mu u_0 \omega s_T. \tag{2.2}$$

![Fig. 1. Incident antiplane wave on a semi-infinite crack in an unbounded elastic solid.](image)