A SUPER-ELEMENT FOR CRACK ANALYSIS

Pin Tong and S. Lasry
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139
tel: 617/253-1986

Using the hybrid concept in the finite-element method [1], a super-element has been constructed for crack analysis and the stress intensity factors can be determined accurately and efficiently.

It has been established that in analyzing the problems involving singularities by the finite element method, the rate of convergence of the approximate solution is usually dictated by the nature of the solution near the point of singularity [2]. The regular so-called high accuracy element (using high order of polynomials as interpolation functions) without properly accounting for the singularities cannot improve the rate of convergence. It is difficult to use the displacement finite element model to account for all the singularities without violating the compatibility conditions. To resolve such difficulties using the hybrid stress model [3], we treated the crack problem by adding a singular stress term in its assumed stress solution for the elements around and near the crack tip. Since the hybrid-stress model [4] separated the assumed displacement field over the element boundaries and the equilibrium stress within each element, there is no difficulty in fulfilling the compatibility conditions. In [5], also using the hybrid concept, a super-element at the crack tip is constructed. In this element, a displacement field is assumed over the element boundaries in terms of the element nodal displacement \( \mathbf{q} \), and using the complex variable technique, the stresses and the displacements in terms of unknown \( \mathbf{z} \) are assumed within the element. These stresses and displacements within the element satisfy the equilibrium equations, stress-displacement relations, as well as the stress free condition at the crack surface. The unknown \( \mathbf{z} \) can be eliminated, and the total strain energy of the element can be expressed in the form of

\[
U = \frac{1}{2} \mathbf{q}^T \mathbf{k} \mathbf{q}
\]

in which \( \mathbf{k} \) is the element stiffness matrix and \( \mathbf{q} \) is the element nodal displacement vector. The stress intensity factors are obtained in the form

\[
k_I = \mathbf{B}_I \mathbf{q}
\]

and

\[
k_{II} = \mathbf{B}_{II} \mathbf{q}
\]

where \( k_I \) and \( k_{II} \) are the stress intensity factors of mode I and mode II, respectively.

The super-element has been tested numerically on the problem with edge crack shown in Fig. 1, and for two different meshes shown in

Int Journ of Fracture 9 (1973)
Fig. 2. Both cases give a stress-intensity factor of 0.793 which agrees exactly with that in [6]. In the case of Fig. 2a, there are 4 regular rectangular elements and one super-element at the crack tip. The CPU time on the IBM 370/165 is 1.7 seconds. In the case of Fig. 2b, there are 18 regular elements and one super-element, and the CPU time is 1.8 seconds.

The use of the complex variable technique developed extensively in [6] leads to an efficient method of programming and enables one to account for the singularities to a high order of accuracy. Therefore the present crack element is much improved from that of [3].

Acknowledgment: This work was supported by AFOSR under Contract No. F44620-72-C-18.

REFERENCES


21 February 1973