An interface integral equation method applied to a crack impinging upon a bimaterial, frictional interface

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Received 15 December 1989; accepted in revised form 28 May 1990

Abstract. A crack impinging upon a frictional, bimaterial interface is studied theoretically. Specifically we consider the problem of an infinitely long, cracked, two-dimensional fiber, which is embedded in an infinite plane with distinct elastic properties. The composite is subjected to tensile loading parallel to the fiber. An interface integral equation method is developed to solve this problem. This method, involving to-be-determined distributions of line forces, reduces the specific problem considered here to four coupled integral equations which are solved numerically. The bimaterial effect appears to be significant with respect to the length of the slip zone along the interface and the interfacial shear stress. However, the blunting of the crack by the frictional interface is virtually independent of the bimaterial effect.

1. Introduction

Fiber-reinforced ceramics, developed for high-temperature applications, are generally made of brittle constituents, which are often coupled by a weakly bonded interface. This interface can allow for friction or mechanical interlocking (as suggested by, among other, Phillips [1] and Prewo and Brennan [2]), although this depends on the fiber preparation and the processing. When such a composite is subjected to tensile loading one of the constituents generally fractures more readily than the other; this can result in cracks that impinge on the interfaces. It is believed that a weakly bonded interface significantly contributes to the composite's resistance to crack propagation, for example by deflecting the crack onto the interface, and thus compensates for the brittle character of the constituents.

In this paper we examine this toughening effect of the interface theoretically by focusing on an idealized two-dimensional problem. Consider an infinitely long, two-dimensional fiber embedded in, though not bonded to, an infinite plane of matrix material, having distinct elastic properties. Consistent with the weak interface often found in brittle-matrix composites, contact across the interface is taken to be described by Coulomb friction. This composite is assumed to have residual stresses, including a compressive stress acting normal to the interfaces. (Such residual stresses arise in the course of processing and are often associated with differential thermal expansions.) Furthermore, the fiber is broken so that a crack impinges normally upon the interface. Of interest is the response of the composite body to a remote, monotonically increasing tension acting parallel to the fiber. Thus, this problem constitutes a generalization of the problem recently studied by Dollar and Steif [3], in which the elastic moduli of the fiber and matrix were assumed to be identical. There it was found that the stresses at the crack-tip are finite, and that the frictional resistance of the interface influences the stress concentration at the crack tip. Here we are attempting to quantify the
effect of modulus difference on this blunting process. Note that we can exchange fiber and matrix moduli and thereby gain insight into a matrix crack impinging on a fiber.

Unfortunately a solution to this problem does not appear to be readily arrived at via existing methods of analysis. There do exist various solution methods for treating micro-mechanical problems in composites. For example, solution methods for media composed of layers with different elastic properties which are perfectly bonded have been developed by Erdogan and Gupta [4, 5] and Gupta [6]. On the other hand, composites consisting of layers with identical elastic properties which are connected by an imperfectly bonded interface that allows for slip have been studied by Steif and Hoysan [7] and Dollar and Steif [3, 8]. None of the available solution techniques, however, readily allows for problems involving composites having material layers with different elastic properties which are connected by an imperfectly bonded interface. (One exception is recent work by Wang, Hui, Lagoudas and Papadopoulos [9] who used a technique akin to Dollar and Steif [3] to treat the specific problem of a semi-infinite crack in one material impinging upon a second material to which it is connected by friction.) The problem of interest here, therefore, requires the development of a new solution technique. In this paper we will present a rather general solution method, which is suitable for the study of composites with constituents having dissimilar elastic moduli and with interfaces with mixed conditions. This method is tested on a number of problems and then applied to the problem of interest here.

2. Description of the problem

The problem we are considering is shown schematically in Fig. 1. An infinite strip, occupying \(-a < x < a\), is sandwiched between two half planes, occupying \(x > a\) and \(x < -a\). The strip and the two half-planes are homogeneous, isotropic and linear elastic, with elasticity

![Fig. 1. Schematic of cracked fiber in infinite plane.](image-url)