Three-strip yield model in mixed-mode fracture

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Abstract. In this paper an alternative model with three yield strips is proposed to represent the crack tip plasticity under a mixed-mode loading. Based on the superdislocation method, the problem can be reduced to several algebraic equations. Numerical results for small-scale yielding are given.

1. Introduction

Representation of the crack tip plasticity by strip yield model has shown its great potential for understanding the plastic deformations around a crack tip [1–5]. The slip plane inclined to the crack plane in terms of dislocation arrays was first studied by Bilby and Swinden [3]. Vitek [4] and Riedel [5] have obtained more comprehensive results for a slip-plane inclined at various angles. Atkinson et al. [6] introduced the idea of a superdislocation to represent the net effect of the dislocation array. Watanabe and Sato [7] developed a discontinuous model to analyze the plasticity near the crack tip by the finite element method. In this paper, a crack with a mixed-mode loading condition and having three yield strips is proposed. Based on the superdislocation method, the basic equations for including crack tip plasticity are derived and numerical results are presented.

2. General analysis

The development given in this paper is in fact motivated by study of the extension of the inclined strip yield model to an interface crack [8], since interface fracture is itself intrinsically mixed-mode. However, as will be described below, some difficulties were met when the inclined strip yield model was used.

For convenience, a Griffith crack with the length of 2a in a homogenous material is considered. A remotely applied stress $T + iS$, where $T$ denotes the stress normal to the crack plane and $S$ the shear stress parallel to the crack plane is assumed (see Fig. 1). It will suffice to describe the location and the strength of the superdislocation on the right tip due to the symmetry. These are given by $l_i(i = 1, 2)$, the distance from the crack tip to the superdislocation, and $b_i(i = 1, 2)$ the strength of the superdislocation. In the following the

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index $i = 1, 2$ denotes the upper half-plane $S^+$ and the lower half-plane $S^-$, respectively. Now introduce the nondimensional quantities as follows:

$$
\begin{align*}
\vec{T} &= \frac{T}{G/(1 + \kappa)} , \\
\vec{S} &= \frac{S}{G/(1 + \kappa)} , \\
\vec{b}_i &= \frac{b_i}{a} , \\
\vec{l}_i &= \frac{l_i}{a}
\end{align*}
$$

(1)

One of the key elements for the superdislocation model is for the cancellation of the crack tip singularity. In the case of small-scale yielding, this condition gives [8]

$$
\frac{\bar{b}_1}{\sqrt{l_1}} (\sin \theta_1/2 + \sin 3\theta_1/2) - \frac{\bar{b}_2}{\sqrt{l_2}} (\sin \theta_2/2 + \sin 3\theta_2/2) = \frac{\sqrt{2\pi}}{3} T ,
$$

(2)

$$
\frac{\bar{b}_1}{\sqrt{l_1}} (\cos \theta_1/2 + 3 \cos 3\theta_1/2) - \frac{\bar{b}_2}{\sqrt{l_2}} (\cos \theta_2/2 + 3 \cos 3\theta_2/2) = \sqrt{2\pi} \bar{S} ,
$$

(3)

where $G$ is shear modulus and $\kappa = 3 - 4\nu$ for plane strain and $(3 - \nu)/(1 + \nu)$ for plane stress. $\theta_1$ and $\theta_2$ denote the angles between the crack plane and slip plane on the upper half-plane $S^+$ and the lower half-plane $S^-$, respectively.

Let us consider a special case where $\theta_1 = -\theta_2 = \cos^{-1}(1/3)$, and the left side of (3) will vanish. This means that the interaction between the crack and dislocations at these angles makes no contributions to the shear stress along the crack plane and, $\bar{b}_1/\sqrt{l_1}$ and $\bar{b}_2/\sqrt{l_2}$ become undetermined. When $\theta_1$ and $-\theta_2$ are not equal to $\cos^{-1}(1/3)$ simultaneously, we can get the solutions of $\bar{b}_1/\sqrt{l_1}$ and $\bar{b}_2/\sqrt{l_2}$ as the function of $\theta_1$ and $\theta_2$, from (2) and (3). For given $-\theta_2 = 30^\circ$ and $-\theta_2 = 80^\circ$, respectively, Figs. 2 and 3 give the results of $\bar{b}_1/\sqrt{l_1}$. It can be found that $\bar{b}_1/\sqrt{l_1}$ varies rapidly when $\theta_1$ approaches $\cos^{-1}(1/3)$. Because the variable $\bar{b}_1/\sqrt{l_1}$ must take positive value, Figs. 2