Wave forces on arrays of floating bodies

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Summary

Previous work on the scattering of an incident wave field by an array of fixed vertical cylinders is extended to calculate the added-mass and damping coefficients for an array of floating axisymmetric bodies. The method is based upon a large spacing approximation where diverging waves are replaced by plane waves. It is shown that, given the scattering and radiation properties of a single body, the interaction effects within an array can be calculated both simply and accurately.

1. Introduction

The continuing development of offshore structures has led to an increasing interest in the hydrodynamic interactions between neighbouring structures due to wave motions. The scattering of an incident wave field by a group of bodies may lead to wave forces on one of the bodies that differ significantly from the forces it would experience if in isolation. Neglecting viscous effects and using linearised water wave theory, a number of authors have computed these interaction effects. Complex body shapes can be handled using finite element or source distribution techniques; however, computations involving multiple bodies can be prohibitively expensive. Fortunately, many offshore structures are supported by fixed or floating axisymmetric elements and the resulting mathematical simplification is of considerable advantage. A number of authors have considered the problem of wave scattering within a group of fixed or floating vertical cylinders. For example, Ohkusu [1] solved the problem using the method of multiple scattering, where each successive scattering event is considered separately, while Matsui and Tamaki [2] used a source distribution method.

Even with the simplification of considering the bodies to be axisymmetric, computation of the interactions within a group of two or three bodies remains quite complex. A significant step in the simplification of the solution procedure was made by Simon [3]. He suggested that a diverging wave, scattered by, or radiated from a body, may be approximated at large distances by a plane wave. The easiest way to envisage this is to consider circular wave crests. For large radii the crests are locally almost straight when considered on an appropriate length scale, namely the wavelength. In addition to the modelling of diverging waves by plane waves, Simon's method neglects the local fields which decay exponentially away from a body.
Simon [3] used his approximate method to calculate the performance of wave-energy devices. Though he had no other solution method available for comparison, Simon did give an order of magnitude estimate of the errors arising from the plane-wave approximation. McIver and Evans [4] (hereinafter referred to as I) used the plane-wave method to make calculations of horizontal wave forces on arrays of fixed, bottom-mounted, surface-piercing vertical cylinders. For this problem an alternative, exact method of solution (within linear theory) is available. In some of the test comparisons in I, the errors in the wave forces calculated using the plane-wave approximation were more than ten per cent when the cylinders were closely spaced. However, it was shown in I that this error could be reduced to about two per cent, or less, by the incorporation of a simple non-plane correction term which involves very little additional effort. In fact, the use of this correction term for horizontal forces is necessary to obtain the same degree of accuracy given by the plane-wave approximation for vertical forces (see Appendix I).

It is a feature of the scattering problem for the fixed vertical cylinder considered in I that there is no specifically load field decaying exponentially with distance from the body. In general such fields are present, but in the plane-wave method their influence on neighbouring bodies is neglected. This has been shown to be valid in two dimensions (e.g. Srokosz and Evans [5]), even when the body spacing is small. One aim of the present work is to test the plane method when local fields are present in the full linear problem. The situation considered is that of a pair of floating docks for which accurate computations have been made by Matsui and Tamaki [2].

The main purpose of the present work is to provide a more rigorous test of the modified plane-wave method as described in Paper I and to show how it can be applied to more general problems. In Section 2 the mathematical problem is formulated whilst in Section 3 the coupling between a single body and the waves is described. The method of solution for the scattering problem is given in Section 4 and the expressions for the added-mass and damping coefficients for an array of bodies are derived in Section 5. The results for the specific problem of a floating dock are presented in Section 6 and a comparison made with the work of Matsui and Tamaki [2].

2. Formulation

Consider a group of $N$ identical bodies floating in water of depth $d$. The geometry of an individual body will not be prescribed as yet, though it will be assumed to be vertically axisymmetric. The coordinate system adopted here is that used in I. Thus, Cartesian coordinates are chosen with the $x$- and $y$-axes in the horizontal plane of the bottom and the $z$-axis directed vertically upwards. A sketch of a horizontal section is given in Fig. 1. The $j^{th}$ body has its centre of cross-section at the point $(x_j, y_j)$; relative to this point the field point has coordinates $(r_j, \theta_j)$, where $\theta_j$ is measured clockwise from the positive $y$-axis. The centre of the $k^{th}$ body has coordinates $(R_{jk}, \alpha_{jk})$ relative to the $j^{th}$ body.

The usual assumptions of linearised water-wave theory are made; i.e. the fluid is taken to be inviscid and incompressible and the motion to be irrotational with particle motions of small amplitude. The fluid motion may then be described by a velocity potential $\Phi(x, y, z, t)$ satisfying Laplace's equation within the fluid and the boundary conditions of no flow through solid boundaries, the linearised free-surface condition and a radiation condition of outgoing waves at large distances. The motion is also assumed to be