Calculation of microstresses in a thick non-symmetric heterogeneous plate

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Abstract. We consider the thermoelastic behaviour of a thick non-symmetric heterogeneous plate and containing in its interior a large number of periodically distributed transverse holes or inclusions. We use the Reissner-Mindlin thermoelastic linear model of thick plates with a known temperature and we distinguish displacements in the upper and lower part of the plate with respect to the middle plane. Due to the structure of the plate, thermal and elastic coefficients are non-uniformly and rapidly oscillating functions of the space variable. Two-Scale Convergence, which is the state of the art in the technique of mathematical homogenization, is used to obtain convergence results and formulas allowing one to calculate the distribution of Microstrains and Microstresses inside the plate when a "macroscopic" behaviour is given. We give an example illustrating the computation of these Microstresses in the case of a symmetric plate.

1. Introduction

1.1. Industrial Origin of the Problem

In this paper we shall give convergence results and an example of a numerical experiment on the thermoelastic behaviour of an heterogeneous plate, the industrial origin of which is to be found in a part of an air-cooler of the turbo-engine of a railway train. Each of these plates are highly heat conducting and transversally perforated by a great number of "tunnels" in which heat exchange is done by circulation of atmospheric air and escape gas. The perforated plates are piled up. Fresh atmospheric air is warmed by circulating one way in one plate while escape gas is cooled by circulating the opposite way in the two next plates (Fig. 1).

Here, we consider only one of these plates. The tunnels inside it can have a rather complicated geometry but are distributed in the interior in order to give a periodic geometry for the structure of the plate (Fig. 2).

We consider holes in this original problem, but the heterogeneity of the plates could be due to one or several different material inclusions without any change in the model.

1.2. Aims and Motivation

We want to study the global elastic behavior of this plate when submitted to high (but known) variations of temperature. This question enters in the framework of more general motivations.

As a matter of fact, the global (or effective or homogenized) behavior of inhomogeneous materials, where the inhomogenities are on a small scale, is of considerable interest in material science, mechanical engineering, and in many technological applications, especially those involving composite materials. The theory of homogenization was created to model and predict the behavior of such materials, when the inhomogenities are on a scale much smaller than the linear dimensions of the system. In these situations, continuum models in which the
rapid oscillation of the material properties, because of the inhomogenities, is smoothed out are sufficient to describe the behavior of the system, and possess the advantage of avoiding extensive numerical computations needed when the small scale behavior is taken fully into account.

Here, we give results allowing one to calculate in a simple way internal strains and stresses inside the generic cell of this periodic structure. They are called Microstrains and Microstresses and are obtained as correctors of the “homogenized” behavior of the plate.

In the second section, we discuss and describe the choice of the model used for thick heterogeneous thermoelastic non-symmetric plates; in the third section, we give the mathematical result of convergence which leads to a homogenized problem which allows us to compute Microstresses; a simple example in the symmetric case is given in a fourth section. Mathematical discussion and detailed proofs are given in appendices.

The model chosen is the Naghdi-Reissner-Mindlin Model of thick plates ([1], [2], [3], [4], a rather exhaustive bibliography can be found in [1]). This problem has already been studied in the case of symmetric holes with respect to the middle plane of the plate [5], [6], [7] and in the non-symmetric case [8], [9] by the author and others [10], [11], [12], from a theoretical and numerical point of view, but we give here new results concerning the same model by using the very recent and powerful homogenization technique of Two-Scale Convergence [13], [14], [15].