The cognitive effects of a mathematics in-service workshop on elementary school teachers

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Abstract. Siegler's rule assessment methodology was used to investigate the cognitive effects of a 32 hour mathematics in-service workshop on 28 elementary school teachers. This paper reports on attempts (a) to assess the levels of cognitive understanding of the basic concepts of proportion, probability, and correlation among elementary teachers, and (b) to change teachers' levels in understanding these concepts by a method of “direct instruction”. Significant improvements were noted in the levels of cognitive development associated with the concept of proportion. Though nonsignificant improvements were noted in the concepts of probability and correlation, all the teachers were assigned to the highest cognitive rule associated with the probability concept at the end of the workshop.

Introduction

Mathematics instruction is a major component in elementary education. If elementary school teachers are to be successful in teaching mathematics, then they need to understand the basic mathematical content that they will be teaching. Unfortunately, many elementary teachers are deficient in the mathematical knowledge that they teach their students (Lacampagne et al., 1988; Post et al., 1988). From a sample of over 200 elementary teachers in a large Midwestern city, a little over 50% could divide 1/3 by 3, and less than 50% could divide 3 by 4/3 (Post et al., 1988).

A remediation program to retrain less knowledgeable teachers in these “basic skills” might be one solution to this problem. However, mathematics educators argue that remediation programs that merely reteach the basic skills do not address the underlying problem, which is a faulty understanding of the concepts and principles behind these basic skills (NCSM, 1978; NCTM, 1980). Ideally, a remediation program would aim at cognitive training in the concepts underlying elementary school mathematics.

One set of fundamental concepts underlying elementary school mathematics are those of proportion, probability and correlation that are identified in the work of Piaget (Inhelder and Piaget, 1958; Siegler, 1981). Although the concepts are fundamental, they are not simple one-variable concepts such as height and weight. These concepts entail the coordination of several variables and thus are sophisticated concepts.
In the case of proportion, the concept is applicable to any situation involving variables a, b, c, and d if the variables satisfy an equality of two ratios such as $a/b = c/d$. An example of such a situation involves a balance scale with two weights in which (a) the weight on the left side of the fulcrum weighs $W_1$ ounces and is $D_1$ inches from the fulcrum, (b) the weight on the right side of the fulcrum weighs $W_2$ ounces and is $D_2$ inches from the fulcrum, and (c) the scale is level. If these variables $W_1$, $D_1$, $W_2$, and $D_2$ satisfy the proportional relationship $W_1/W_2 = D_2/D_1$, then the concept of proportion is applicable to this situation. A proportional relationship allows one to make correct predictions regarding the value of one of the variables if one knows the values of the other three variables.

Children with inadequate understanding of these concepts will often ignore one of the relevant variables and make judgments based on a proper subset of the relevant variables. Thus, such a child will say that a balance scale is level if a 5 ounce weight is placed 2 inches from the fulcrum on the left side of the scale and a 5 ounce weight is placed 4 inches from the fulcrum on the right side, because the two weights are equal. To such an individual, the distance of either weight from the fulcrum is irrelevant.

As children develop more sophisticated understanding of multiple variable concepts such as proportion and probability, they learn to encode all of the relevant variables and they learn how the variables are to be coordinated. Inhelder and Piaget contended that adolescents and adults will have well-developed concepts of proportionality, probability and correlation only after they can coordinate several variables into an integrated system.

Inhelder and Piaget used clinical interviews to assess levels of understanding of adolescents regarding these three fundamental concepts. Though the interviews provide some corroboration for their theory, that theory lacks precision in terms of differentiating among levels of understanding of the concepts. That precision can be garnered by using a detailed information processing model of cognitive processing.

A clear analysis of an intelligence task was provided by Siegler (1976) regarding the Inhelder-Piaget balance task. He used information processing methods to examine the concept of proportionality. Siegler posited that there will be developmental differences with respect to the balance task, because children at different ages will demonstrate different levels of understanding of the concept by employing different cognitive rules in their responses to the task. For his task items, Siegler posited four cognitive rules, each of which indicates a different level of understanding of the proportionality concept. These cognitive rules are explained in greater detail later in this article.

Siegler's rule-based assessment method, though still debatable, makes possible precise testing of Inhelder and Piaget's ideas regarding the growth of sophisticated logical ideas in children, adolescents and adults. This attribute of "testability" is one of the hallmarks of scientific theorizing (Popper, 1963). The