On displacement-based theories of sandwich plates with soft core

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Abstract. The paper is concerned with a family of refined models of elastic sandwich plates with soft core. Construction of this family is based upon the kinematic assumptions of Dundrová, Kovařík and Šlapáč [4]. The model energy-consistent with this hypothesis turns out to be nonelliptic. However, this model makes it possible to find a generalization of Hoff’s [7] model in which transverse normal deformations of the core are partly considered. A proof is given that both this and Hoff’s model is correctly stated irrespective of the choice of fields that describe the angles of rotation of the plate cross-sections. On the other hand, in the model of Reissner [18] this flexibility is lost and only one choice of fields standing for rotations is admissible – namely that in which the assumptions of the Lax-Milgram lemma are fulfilled.

1. Introduction

The first successful attempt to describe deformations of sandwich plates with soft core is due to Reissner [18]. According to this concept the equations of transversely homogeneous moderately thick plates [17] can model deformations of the sandwich plates, provided that the bending and shearing stiffnesses are appropriately modified, see also Plantema [14], Ganowicz [6], Kaczkowski [8]; for further references see Noor and Burton [13]. The Reissner [18] model has been generalized by Hoff [7] for the case of plates in which the bending energy of face-plates cannot be neglected, see also Wachowiak and Wilde [21], Stamm and Witte [20]. Effects due to bending the core have been considered in Eringen [5]; transverse shearing deformations of the facings have been described by Yu [22]. Special attention should be focussed on the approach of Dundrová, Kovařík and Šlapáč [4], suitable for plates with soft core. In this approach, bending of the face-plates as well as transverse normal and transverse shear deformations of the core are taken into account. Moreover, the stresses in the core associated with the assumed kinematics satisfy the equilibrium equations identically, which distinguishes this approach from the others.

In [4] the equilibrium equations of the plate model are derived by conventional averaging of the equilibrium equations across the thickness, which is called a Bollé-Mindlin manner, cf. [19]. In the present paper the construction of the model is based upon the virtual work equation which is the Lax-Milgram equation for the three-dimensional elasticity problem of the plate. According to this method the kinematic hypothesis (DKS) of Dundrová, Kovařík and Šlapáč leads to a refined plate model in which the strain energy consists of five components standing for: energy of in-plane deformations, energy of bending of the face-plates, overall bending energy of the sandwich and energy due to transverse shear strains and transverse normal strains of the core. The energy thus defined is positive definite, which assures that the solution is unique, provided it exists. It occurs, however, that for the natural choice of the space $V$ of kinematically admissible fields the bilinear form defining the energy is not $V$-elliptic. The problem of constructing the space in which the solution would exist remains open.
The proposed sandwich plate model energy-consistent with the (DKS) kinematic hypothesis is a natural departure point for deriving models of simpler form. Upon neglecting the less important terms in the expression standing for the energy of the transverse normal deformations of the core, one arrives at a new refined model of the Hoff type. The remaining terms standing for this energy introduce some slight modifications to the genuine theory of Hoff [7]. It is shown that this amended model is $V$-elliptic and thus correctly posed. Discarding the terms that stand for the energy of transverse normal deformations leads to the model of Hoff. Neglecting the energy of bending of the face-plates reduces the latter model to the theory of Reissner [18].

One of the aims of this paper is to show that in the models of Hoff type the energy of the facings makes the model stable in the sense that the model is correctly posed irrespective of the choice of kinematic fields allowed within the theory. Moreover, the constant of ellipticity does not depend on the relative thickness of the plate. Neglect of this energy results in the loss of such arbitrariness. Therefore, in the model of Reissner [17, 18] there is only one set of unknowns which assures $V$-ellipticity of the problem.

The summation convention is used throughout the paper. Greek indices take values 1, 2. Latin indices run over 1, 2, 3.

2. Problem formulation

The subject of consideration is an elastic sandwich plate of constant thickness $2h$, composed of a core layer of thickness $2c$ and of two external layers (face-plates) of thickness $d$, see Fig. 1. The plate is symmetric with respect to its middle plane $\Omega$. The domain $\Omega$ is parametrized by Cartesian coordinates $(x_\alpha)$. The axis $x_3 = z$ is perpendicular to the middle plane. Thus the layers of the plate occupy the following domains:

![Fig. 1. Geometry of the sandwich plate.](image)