Principal Resonance of a Nonlinear System with Two-Frequency Parametric and Self-Excitations

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Abstract. The principal resonance of a single-degree-of-freedom system with two-frequency parametric and self-excitations is investigated. In particular, the case in which the parametric excitation terms with close frequencies is examined. The method of multiple scales is used to determine the equations of modulation of amplitude and phase. Qualitative analyses are employed to study the behaviour of steady state responses, limit cycle responses and 2-torus responses, including their stability and bifurcation. The effects of damping, detuning, and magnitudes of self-excitation and parametric excitations are analyzed. The theoretical analyses are verified by numerical integration results of the governing equation and the modulation equations.

Key words: Principal resonance, nonlinear system.

1. Introduction

There are many phenomena in which parametric and self-excited vibrations interact with one another. Examples are flow-induced vibrations and vibrations in rotor systems. Moreover, some parametric excitations may contain two or more periodic components whose frequencies are not simple multiples of one another. In particular, complex dynamic behaviour has been observed when some of the excitation frequencies are close to one another.

The responses of single- or multi-degree-of-freedom nonlinear systems to parametric excitations with single frequency have been investigated in [1–9]. Yano [1] investigated some nonlinear models described by typical excitation mechanisms, in which the self-excitation was idealized by a nonlinear resistance of van der Pol type and the parametric excitation contains quadratic and cubic nonlinearities. Both principal and fundamental parametric resonances were discussed in comparison to the analysis of linear modelling. Kotera and Yano [2] dealt with a van der Pol–Mathieu type equation with cubic nonlinearities in the restoring force, which described a beam subjected to a periodic axial force and simultaneously to a flow-induced vibration. Periodic solutions in the regions of principal and fundamental parametric resonances were approximated by the sum of two frequency components and a stability criterion for periodic solutions was established. Kojima et al. [3] considered second order superharmonic and one-half order subharmonic resonances in the vibration of a beam with a mass subjected to an alternating electromagnetic force. In the monograph by Schmidt and Tondl [4] there are some discussions on the interactions between parametric and self-excitations. Nayfeh [5] investigated the response of a multi-degree-of-freedom system with quadratic nonlinearities undergoing a principal parametric resonance. Nayfeh and

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Zavodney [6] studied the response of a two-degree-of-freedom system with quadratic nonlinearities undergoing a combination parametric resonance. In [7, 8] Zavodney and Nayfeh et al. analyzed the responses of fundamental and principal resonances for a one-degree-of-freedom system with quadratic and cubic nonlinearities. Complicated dynamic behaviour, such as phenomena of jump, quenching, modal saturation, Hopf bifurcation, period-multiplying and demultiplying bifurcations, and chaos, etc. were exhibited through numerical computations. Moreover, multiple steady state responses and other periodic responses which depend on initial conditions were also found by numerical integration. In the work of Chen and Langford [9], a generic classification of steady state responses for the nonlinear Mathieu equation was investigated by qualitative methods of catastrophe theory and equivariant singularity theory. Although various dynamic behaviour of responses have been encountered in numerical integration results of nonlinear parametric vibrations, most of the theoretical analyses quoted thus far were mainly concerned with steady state responses. Hence, it is important to make more studies on nonsteady responses analytically.

The responses of linear and nonlinear systems subjected to multi-frequency parametric excitations have been investigated in [10–14]. Nayfeh [10, 11] investigated the responses of combination and principal parametric resonances for linear multi-degree-of-freedom systems. Nayfeh and Jebril [12] analyzed the responses of a two-degree-of-freedom system with quadratic and cubic nonlinearities subjected to multi-frequency parametric excitations. The steady state responses and their stability were determined. Bifurcations and nonsteady state responses were obtained using numerical integration. Barr [13] gave a review on the stability of nonlinear parametric vibrations, including some discussion on multi-frequency parametric excitations. Moreover, Davis and Rosenblat [14] studied the transition curves between stable and unstable regions in the parameter plane for a generalized linear Mathieu–Hill equation with two-frequency parametric excitations. However, very limited attention has been paid to the study of the responses of systems with multi-frequency parametric excitations which have nearly equal frequencies. As systems with excitations of nearly equal frequencies possess beating phenomena, it is extremely important to seek a better understanding of the mechanisms involved.

The present work is therefore concerned with the response of a single-degree-of-freedom nonlinear system containing two-frequency parametric and self-excitations undergoing a principal parametric resonance. In particular, the case in which the parametric excitation terms with close frequencies is examined. The system is governed by the following equation

\[ \ddot{u} + 2\varepsilon (\mu + \nu u^2) \dot{u} + (\omega^2 + \varepsilon \alpha \cos \Omega_1 t + \varepsilon \beta \cos \Omega_2 t)u = 0, \]  

where \( \varepsilon \) is a small parameter, \( \mu, \nu \neq 0 \) are constants, \( \alpha, \beta, \omega, \Omega_1 \) and \( \Omega_2 \) are positive constants. The method of multiple scales is used to construct a first-order uniform expansion of the solution of equation (1). After the amplitude and phase modulation equations are derived, qualitative analysis and asymptotic expansion techniques are employed to predict the existence of steady state responses, limit cycle responses and 2-torus responses of the amplitude. Stability and bifurcations are also investigated. The effects of damping, magnitudes of self- and parametric excitations, and detuning the parametric resonances are discussed. The perturbation solutions and their qualitative behaviours are verified by numerical integration results of the governing equation (1) and the modulation equations, using the fourth order Runge–Kutta algorithm.