Quasiperiodic Oscillations in Robot Dynamics

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Abstract. Delayed robot systems, even of low degree of freedom, can produce phenomena which are well understood in the theory of nonlinear dynamical systems, but hardly ever occur in simple mechanical models. To illustrate this, we analyze the delayed positioning of a single degree of freedom robot arm which leads to an infinite dimensional dynamical system. Restricting the dynamics to a four dimensional center manifold, we show that the system undergoes a codimension two Hopf bifurcation for an infinite set of parameter values. This provides a mechanism for the creation of two-tori in the phase space and gives a theoretical explanation for self-excited quasiperiodic oscillations of force controlled robots. We also compare our results with experimental data.

Key words: Codimension two bifurcation, force control, sampling, time delay.

1. Introduction

It is well known that force controlled robots tend to lose robustness or even stability depending on the environment they are in contact with. Since force disturbances, like the Coulomb friction, may cause large fluctuation in the contact force between the robot end effector and the environment (or workpiece), the application of great proportional control gains is necessary to reduce this error. However, large gains often cause instability. There are several proposed explanations for this type of instability at high gains, including, e.g., Craig [3], Asada and Slotine [2], and Vischer and Khatib [18]. In contrast to these studies, the experimental evidence presented in this paper suggests that the most important cause of instability is the effect of sampling time and the related delay in digital control. Without modeling these effects, one can neither give an accurate description of the loss of stability nor explain the appearance of nonlinear oscillations.

Continuous models, like those of An et al. [1] or Eppinger and Seering [4], can explain why force controlled robots tend to lose their stability more easily than their position controlled counterparts. At the same time, these models leave us without any conclusion for the optimal choice of sampling times. Whitney [19, 20] presents some analytical stability results for digital force control, but his models are too simple to account for the contact between the two basic vibratory systems: the soft force sensor and the environment. The simulation results of Kuno et al. [10] emphasize the possible destabilizing effect of sampling showing instability in force controlled robot systems at a sampling time near 0.1 ms. However, in many cases, e.g., for the experimental hybrid position/force controlled Newcastle robot studied in this paper, the sampling delay is substantially higher than this value. Only models including the effect of
sampling were successful to explain the often unexpected stability properties of the Newcastle robot, which was originally designed for turbine blade polishing (Steven and Hewit [16]).

In this paper we study two robot systems which are closely related to the Newcastle robot mentioned above. First, we briefly discuss some experiments on a kinematically simplified version of the robot (single-axis force control in a limited domain of the workspace). We present stability charts constructed from experimental data and compare them with results obtained from a linear stability analysis of the system. We also discuss a truly nonlinear phenomenon, the occurrence of quasiperiodic oscillations with two or three frequencies slightly above the stability limit. We then propose a simple model which turns out to capture some of these nonlinear effects. This model contains time delay and admits a stability chart which has the same structure as that of the Newcastle robot. We study our infinite dimensional model using center manifold reduction, normal forms, and the methods of bifurcation theory to obtain closed form analytical results. These results indicate the existence of a two dimensional invariant torus and one or two limit cycles within a four dimensional center manifold in the infinite dimensional phase space of the model. These invariant manifolds in the model system explain the occurrence of quasiperiodic and periodic oscillations in the underlying physical system. We also present numerical results which confirm our analytical calculations. Finally, we discuss how more sophisticated modeling could explain the occurrence of stable three-frequency motions in real robot structures.

2. Stability Experiments on Digital Force Control

In order to obtain clearly arranged experimental results, we used a single-axis force control implementation of the hybrid controlled Newcastle robot and studied it only near a certain point of the workspace (see Stépán et al. [14] for details). Although the corresponding mechanical model describes only one DOF of the robot, it still has two mechanical degrees of freedom (see Figure 1) corresponding to the two general coordinates $q_1$ and $q_2$. Note that $q_1$ is the displacement of the first block relative to the second. The coordinates are zero when the spring forces are zero. The control force is denoted by $Q$, $m$ refers to the mass, and $b$ and $k$ stand for the damping factor and stiffness, respectively. The subscripts $r$, $s$, and $e$ refer to the robot manipulator, force sensor, and environment, respectively.

In practice, the manipulator inertia $m_r$ is much greater than the inertia of the sensor $m_s$. The sensor damping $b_s$ is also small compared to the manipulator damping $b_r$. The force sensor is soft (i.e., $k_s$ is small) as suggested by Whitney [20] or Craig [3]. The dynamic behavior of a force controlled robot becomes fairly complicated when the environment is also an oscillatory system (see Figure 1) and this subsystem has a relatively low natural frequency $\sqrt{k_e/m_e}$ and slight damping $b_e$. Since the force sensor is in contact with the environment, the two blocks representing them appear through the joint inertia $m_s + m_e$ in the equations of motion. We