A HAMILTONIAN THEORY FOR AN ELASTIC EARTH: SECULAR ROTATIONAL ACCELERATION

JUAN GETINO*

Instituto de Astronomía y Geodesia, Facultad de Ciencias Matemáticas, Universidad Complutense, Ciudad Universitaria, 28040 Madrid, Spain

and

JOSÉ M. FERRÁNDIZ

Departamento de Matemática Aplicada a la Ingeniería, E.T.S. de Ingenieros Industriales, 47011 Valladolid, Spain

(Received 22 March, 1991; in final form 25 September, 1991)

Abstract. In this article, our previous Hamiltonian theory for the rotation of an Earth whose elastic mantle is deformed by rotation and linsolar attraction is applied to the study of the secular acceleration of the Earth's rotation. Since it is a result of the inelasticity, the theory is extended to include a phase lag. So, we obtain, in a theoretical way, a value of $-5.6 \times 10^{-22}$ rd sec$^{-2}$, which agrees perfectly with the latest observational results.

Key words: Earth tides, Earth rotation, secular acceleration.

1. Introduction

This article is the last of a set of four dedicated to the study, in a Hamiltonian framework, of the rotational motion of an Earth whose elastic mantle is deformed by rotation and gravitational attraction of the Moon and the Sun. In the first of these (Getino and Ferrándiz, 1990), two canonical systems of variables suitable for the problem were built up, which were called Euler and Andoyer's 'elastic' variables respectively, and the kinetic rotational energy was obtained in those sets. In the second (Getino and Ferrándiz, 1991a), the expression of the elastic deformation energy in those variables was given, and a numerical integration for a particular Earth Model, Takeuchi's Model 2 (1950), was performed, computing the value of different coefficients that appear in the said energies. Although the Earth Model used is old, it is sufficient to obtain preliminary numerical results, and this choice is not relevant in the theory's present state of development. In the third paper (Getino and Ferrándiz, 1991b), the approximate analytical integration of the system is performed by means of a Lie series perturbation method; the numerical tables of the periodic perturbations corresponding to the nutation in obliquity and longitude are displayed, and, as for the secular effects, a theoretical value of 457 days is obtained for the Chandler period.

In this paper we address the secular rotational acceleration of the Earth. This phenomenon has been observed for some time and, with the help of highly accurate observation techniques, it is possible to obtain very precise experimental values.

* Permanent address: Departamento de Matemática Aplicada Fundamental, Facultad de Ciencias, 47005 Valladolid, Spain.
Recently, Bursa (1987) developed an approach based on the variation of the angular moment, which provides values of the deceleration in good agreement with the results of the observations. Here we tackle the problem in a different way, applying the Hamiltonian theory for the rotational motion of a deformable Earth described in the three above-mentioned papers. It has been extended by considering that the Earth behaves as an inelastic body, i.e. there is a delay in the time of maximum tidal response, the resulting phase lag being responsible for the secular deceleration. The value of this deceleration obtained theoretically is $5.6 \times 10^{-22}$ rd sec$^{-2}$, which agrees perfectly with the observational data.

The main interest of our approach lies in the fact that, under the hypothesis of an Earth with an elastic mantle, from only a Hamiltonian formulation we can obtain, in a clear and direct way, the solutions for the most important problems of the Earth's rotation: secular and periodic perturbations of the nutations, Chandler period and secular acceleration.

2. Statement of the Problem

2.1. HAMILTONIAN OF THE SYSTEM

Let us consider the problem of an Earth whose elastic mantle is deformed by rotation - centrifugal deformation - and lunisolar attraction - tidal deformation - taking the Moon and Sun as point masses of known orbit. In Getino and Ferrándiz (1991b) the complete Hamiltonian corresponding to this system is described. According to those results, it can be cast as:

$$H = T_0 + T_r + T_E + E_t + E_r + V_2 + \sum_{n=3}^{\infty} V_n + V_T + V_r$$

The principal contribution to the secular acceleration, that is the object of this paper, arises from the tidal potential, $V_T$, so that it is sufficient to take a simplified Hamiltonian as follows:

$$H = T_0 + V_T,$$  \hspace{1cm} (1)

leaving aside for a later section the study of the effect of the remaining terms.

In Equation (1) $T_0$ is the rotation energy without deformation, which is expressed as follows:

$$T_0 = \frac{1}{4} M^2 \left( \frac{1}{A} + \frac{1}{B} \right) + \frac{1}{4} N^2 \left( \frac{2}{C} - \frac{1}{A} - \frac{1}{B} \right) +$$

$$+ \frac{1}{4} \left( M^2 - N^2 \right) \left( \frac{1}{B} - \frac{1}{A} \right) \cos 2\nu,$$  \hspace{1cm} (2)

in the set $(M, N, \Lambda, \mu, \nu, \lambda)$ of elastic Andoyer variables introduced by Getino and Ferrándiz (1990). Although the moments of inertia $A, B, C$, are defined as the moments in absence of deformation, $A_0, B_0, C_0$, plus the increase due to the