On the propagation of generalized transverse waves in heat-conducting elastic materials

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ABSTRACT

A generalized transverse wave is a propagating acceleration discontinuity on which the temperature and the entropy, together with their gradients, are continuous. In a heat-conducting elastic material the propagation and growth of such waves are uninfluenced by thermomechanical interaction. It is shown in this paper that in any given plane there is at least one direction in which a generalized transverse wave may propagate, and the existence is also proved of at least one direction in which a pair of generalized transverse waves may travel. Necessary and sufficient conditions are established for the speeds of propagation of these waves to be real. Relationships between transverse and generalized transverse waves are also studied, and in the last two sections of the paper the directions in which generalized transverse waves may propagate in an isotropic heat-conducting elastic material are systematically worked out and classified.

RÉSUMÉ

Une onde transversale généralisée est une discontinuité d'accélération en train de se propager sur laquelle la température, l'entropie ainsi que leurs dérivées sont continues. La propagation et le développement de telles ondes dans des matières conductrices de chaleur, ne sont pas affectées par interaction thermo-mécanique. On démontre qu'il existe au moins une direction dans tout plan donné dans laquelle une onde transversale généralisée peut se propager, et on prouve aussi qu'il existe au moins une direction dans laquelle une paire d'ondes transversales généralisées peuvent se propager. On détermine quelles sont les conditions nécessaires et suffisantes pour que les vitesses de propagation soient réelles. Les relations entre les ondes transversales et transversales généralisées sont aussi étudiées, et dans les deux dernières sections on obtient et classe systématiquement les directions dans lesquelles les ondes transversales généralisées peuvent se propager dans une matière élastique, isotrope, conductrice de chaleur.

1. Introduction

In three recently published papers Chen [1], Dunwoody [2] and the present writers [3] have independently initiated studies of acceleration discontinuities in heat-conducting materials on which no jump occurs in either the temperature gradient or the entropy gradient. The term generalized transverse wave, applied to such discontinuities in [3, section 7], alludes to the fact that the characteristic relationship between the wave normal and the acceleration amplitude is not orthogonality, as in the case of a transverse wave, but conjugacy with respect to a material properties tensor, namely the temperature coefficient of stress. Additionally, generalized transverse waves in heat-conducting elastic materials have physical significance inasmuch as they are free from thermo-elastic interaction: neither the heat-conducting properties of the material nor its ability to deform in response to changes of temperature have any
influence on the propagation or growth of such waves [3, section 7]. The generalized transverse wave concept may therefore be said to single out a particular type of acceleration wave on the basis of qualities which, unlike the purely geometrical relationships characterizing longitudinal and transverse waves, are intrinsic to the transmitting material 1.

The above-mentioned papers deal rather briefly with different aspects of the behaviour of generalized transverse waves. Chen's work [1] is mainly concerned with principal waves in isotropic heat-conducting elastic materials which are either definite conductors of heat or non-conductors, while Dunwoody's paper [2] relates to a more general class of materials with memory, again with restriction to definite heat conductors and non-conductors. Chadwick and Currie [3, section 7], like Chen, have considered acceleration waves propagating in heat-conducting elastic materials, but their results place no limitation on either the symmetry of the material or the direction of the wave normal, and the constraints on thermal conduction accepted in [1] and [2] are considerably relaxed. It now seems desirable to discuss with greater thoroughness the properties of generalized transverse waves, connecting and extending what has already been learned, and this is our aim in the present paper. We choose, as in [3], to work within the frame of thermoelasticity theory, but the analysis developed in the body of the paper extends in a straightforward manner to heat-conducting materials with memory.

In section 2 we assemble basic results from our earlier paper [3] before proving, in section 3, two theorems on the existence of generalized transverse waves in general heat-conducting elastic materials. It is shown first that every plane contains at least one direction in which a generalized transverse wave may propagate, and secondly that there is at least one direction in which a pair of generalized transverse waves with orthogonal amplitudes may travel. For wave propagation in these directions to be possible the appropriate wave speeds must be real, and necessary and sufficient conditions for the satisfaction of this requirement are given. We also prove in section 3 that a generalized transverse wave is transverse if it propagates along a principal axis of the temperature coefficient of stress and, conversely, that a transverse wave proceeding in such a principal direction is usually transverse in the generalized sense as well.

In the remainder of the paper the discussion is particularized to isotropic heat-conducting elastic materials, and in this case we find it possible to describe much more explicitly than in section 3 the directions in which generalized transverse waves may propagate. The situation regarding principal waves, exposed in section 4, is simple: a non-longitudinal principal wave is usually transverse in both the ordinary and generalized senses. Thus, if principal waves in isotropic materials are regarded as having an archetypal role in the theory of waves propagating in heat-conducting elastic materials in general, the intrinsic characterization of transversality explored in this paper is on an equal footing with the conventional geometrical definition. Finally, in section 5, we determine the possible non-principal directions which may be followed by single generalized transverse waves, by pairs of such waves, and by single waves and pairs which are transverse in both senses.

The results established in this paper which apply exclusively to generalized transverse waves each have a direct analogue for transverse waves, and those results which refer to both transverse and generalized transverse waves do so in a reciprocal manner. Collectively, therefore, our findings demonstrate that in heat-conducting elastic materials generalized transverse waves occur as frequently as transverse waves.

1) We have described elsewhere [4] a natural extension of this viewpoint which includes quasi-longitudinal as well as generalized transverse waves.