Numerical Derivation of Forced Nutation Terms for a Rigid Earth

J. Schastok\textsuperscript{1,2}, M. Soffel\textsuperscript{2}, H. Ruder\textsuperscript{2}

\textsuperscript{1}Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, CA 91109, USA

\textsuperscript{2}University of Tübingen
Lehrstuhl für Theoretische Astrophysik
Auf der Morgenstelle 10
7400 Tübingen, W.Germany

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Abstract

We compare the results of a numerical integration of the Euler equations for a rigid Earth model covering a time span of 250 years with Kinoshita’s theory for the forced nutations and with a new nutation series by Kinoshita and Souchay. We also present numerical corrections to some of the analytically derived nutation terms.

Keywords: forced nutation, rotation of the Earth, numerical integration

Introduction

Since the accuracy of present VLBI-determinations of the nutation angles of the Earth is greater than the uncertainties of the currently adopted nutation series for an elastic Earth model (Wahr 1981) for which the gyroscopic motion of a rigid model Earth serves as basis one faces the necessity to improve Kinoshita’s (1977) results for the nutations of a rigid Earth. Improvements in this direction can be found in Kinoshita (1988) and especially in Kinoshita and Souchay (1989), where calculations of nutation terms are carried out with higher accuracy and including effects that are not considered in the old nutation series.

Here, we report on new results based upon a purely numerical treatment of the rotation of a rigid model Earth including torques by Moon and Sun that were evaluated using the ephemeris DE102 of the JPL. Our nutation series were
derived from a numerical integration of the Euler equations by means of least square fit methods considering an integration time span of about 250 years.

First we present our corrections to the 18.6 and 9.3 year terms as given by Kinoshita's (1977) analytical theory for the nutations of rigid Earth, which can directly be compared with the corrections given in Kinoshita (1988). We then present small deviations we found from the new (semi-) analytically produced nutation series by Kinoshita and Souchay (1989).

Numerical Procedure

For the sake of consistency with the used ephemerides we adopted dynamical parameters as GM - values for Earth, Moon and Sun, mass multipole moments and the equatorial radius of the Earth from DE102 (see Newhall et al. 1983):

\[
\begin{align*}
GM(\text{Sun}) &= (0.01720209895)^2 \text{au}^3/d^2 \\
GM(\text{Sun})/GM(\text{Earth} + \text{Moon}) &= 328900.53 \\
GM(\text{Earth})/GM(\text{Moon}) &= 81.3007 \\
J_2 &= 1082.637 \times 10^{-6} \\
J_3 &= -2.541 \times 10^{-6} \\
J_4 &= -1.618 \times 10^{-6} \\
R &= 6378.156 \text{ km} \\
AU &= 149597870.68 \text{ km}.
\end{align*}
\]

The moments of inertia for the Earth's figure are computed using the dynamical ellipticity given by Kubo and Fukushima (1987)

\[
H = (C - A)/C = 0.0032739935.
\]

With the assumption of axial symmetry we get the moments of inertia:

\[
\begin{align*}
C &= J_2 R^2/H \\
A &= B = C - J_2 R^2.
\end{align*}
\]

The integration was carried out with respect to mean ecliptic and equinox of the epoch J2000. For this purpose DE102 was transformed into the mean equatorial system of the epoch J2000 by use of the rotation matrix given by Newhall et al.