EXTERNAL PERTURBATIONS ON DISTANT PLANETARY ORBITS AND OBJECTS IN THE KUIPER DISK

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Abstract. We investigate the potential importance of molecular cloud and stellar perturbations on the orbits of Pluto and more distant (hypothetical) planets up to 500 AU from the Sun. It is found that stellar and molecular cloud-core perturbations are of roughly equal importance. It also is found that the likelihood of substantial perturbations on Pluto is not insignificant, and that numerous substantial stellar and molecular cloud perturbations are likely to have influenced the orbits of any planets beyond ~200 AU. These perturbations may contribute to a prevalence of moderate eccentricities and inclinations for planets beyond the orbit of Neptune, and may be a characteristic of distant planetary orbits in other solar systems. Given the recent discovery of chaotic behavior in Pluto's orbit (Sussman and Wisdom 1988), the effects of external perturbations on the long-term stability of Pluto's orbit warrant continued study.

1. Introduction

Stellar and molecular cloud perturbations are known to control the distribution of orbits in both the outer and inner Oort Clouds (e.g., Bailey et al. 1986). Impulses caused by close encounters with these objects are important in the comet cloud primarily due to the great distance of the cloud from its central field source, the Sun.

This brief communication assesses the potential importance of stellar and molecular perturbations on planetary and comet orbits with semi-major axis \( a < 500 \) AU.

The issue of perturbations by passing stars and other 'external' influences on planetary orbits has become more important since Sussman and Wisdom (1988) found that the orbit of Pluto is chaotic with a Lyapunov exponent corresponding to an \( e \)-folding divergence timescale of \( 2 \times 10^6 \) years. External perturbations by Pluto and more distant (hypothetical) planets due to close encounters with stars can exceed the effects of the Jovian planets by an order of magnitude; the effects of external perturbations were not considered by Sussman and Wisdom.

2. Model

To investigate the importance of external perturbations on planetary orbits, we shall first calculate the number of encounters a hypothetical planet at given heliocentric distance suffers due to stars or other objects capable of making substantial perturbations. By substantial, I refer to a perturbation comparable to that Jupiter has on Pluto.

Application of kinetic theory dictates that over the age of the solar system, the
number of encounters $N_e$ in which a perturbation of magnitude $\alpha$ occurs, is

$$N_e(\alpha) = \sum_i n_i \pi R_{i,p}^2(\alpha) v_i T_{ss}$$  \hspace{1cm} (1)$$

where $n_i$ is the space density of perturbers of type $i$, $v_i$ is the mean velocity at which perturbers of type $i$ pass the solar system, $R_{i,p}$ is the impact parameter for a perturbation of magnitude $\alpha$, and $T_{ss}$ is the age of the solar system. This formalism assumes the standard particle-in-a-box analogy and neglects the effects of gravitational focusing (which may easily be shown to be small for stellar encounters with the solar system).

I stress that $R_{i,p}$ is the distance at which a **differential** acceleration (between the planet and the sun) of specified magnitude occurs. We normalize this differential acceleration by the solar gravitational acceleration on the planet of interest, and denote the dimensionless relative acceleration as $\alpha$:

$$\alpha_i = \frac{a_{i,p} - a_{i,\odot}}{a_{\odot,p}}$$  \hspace{1cm} (2)$$

Here $\alpha_i,x$ is the gravitational acceleration of the intruding object $i$, on object $x$; the subscript $p$ refers to the planet, $i$ to the intruder, and $\odot$ to the Sun. Substituting for $a$ according to Newtonian gravity and rearranging gives

$$\alpha_i = \mu R_{i,\odot}^2 \left( \frac{R_{i,\odot}^2 - R_{i,p}^2}{R_{i,\odot}^2 R_{i,p}^2} \right)$$  \hspace{1cm} (3)$$

where $R_{i,\odot}$ is the perturber-Sun distance, $R_{i,p}$ is the perturber-planet distance, $R_{p,\odot}$ is the planet-Sun distance, and $\mu$ is the mass ratio of the perturber to the Sun. Of course, $R_{i,\odot} = R_{i,p} + R_{p,\odot}$; the limit $R_{p,\odot} \ll R_{i,p}$ reduces this formalism to the familiar inverse-cube ratio for differential perturbations. In the calculations below, such a limit does not always obtain. Therefore, for a specified value of $\alpha$, we solve Equation (3) exactly for $R_{i,p}$ and insert the result into Equation (1) to calculate the encounter statistics as a function of perturber mass.

For an encounter to be important, it must occur on an impulsive timescale. This requirement must obtain to prevent the perturbation from being averaged out by orbital motion. This in turn requires the perturbing encounter be short compared to the planetary orbit. An ensemble average over many encounters within distance $R_i$ gives a mean encounter duration $t_{enc}$ of

$$t_{enc} = \frac{4}{\pi} \left( \frac{R_i}{v_i} \right)$$  \hspace{1cm} (4)$$

For an encounter to strongly perturb a planetary orbit, we therefore require the encounter timescale be such that $t_{enc} < T/2\pi$, where $T$ is the planetary orbit period. In summing $N_e$, I therefore only keep those encounters with $T_{enc} < 39$ years for Pluto and $T_{enc} < 450$ years for a hypothetical planet at 200 AU.